## Comparative Study of Transport Models for Bipolar Switching in Memristors

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## Memristors - Explosive Recent Interest

HP group of Stan Williams reported hysteretic switching behavior in Pt/TiO/Pt structures (Nature 2008)
As of midnight: 529 citations



## Switching by Channel Formation

- TMO systems are inherently inhomogeneous
- Switching mechanism: channel formation by oxygen vacancy migration


Becker et al PRL ‘02

## TM transport evidence for channel formation


b)

c)

d)


Chae (2008)

## Electronic Channel Formation model


(b)

(e)

## Top Electrode



Middle Domains


Bottom Domains
Bottom Electrode

## Channel Model of <br> Rozenberg, Sanchez, Levy et al

a

b
initial oxygen vacancy density profile

"formed" oxygen vacancy density profile




## Channel Model of <br> Rozenberg, Sanchez, Levy et al



|  | Rozenberg et al (2010) |
| :--- | :--- |
| Vacancies | increase resistance |
| Boundary layer ("A") | present |
| Electron dynamics | implicit |
| Pile-up next to interface | vacancies |
| Cause of switching | vacancy bump inside <br> interface moves from <br> next to electrode into bulk |

Metallic conduction: not intended for binary oxide systems

## Channel lateral dynamics model of Kittl group



Limited utility:

- for narrow devices
- in light of direct imaging


## Switching by Front Dynamics



Thermal image


O vacancy density from X-ray fluorescence

Switching may involve sample-wide front/wall moving

## HP Modeling: Pickett et al.

$$
\text { Switching }=\text { Movement of wall of O vacancy-rich region }
$$



Phenomenology: variable w, domain wall location between high $R$ and low $R$ region moves

Double exponential dynamics is introduced to reproduce data, lacking clear motivation:
Undoped:


$$
\dot{w}=f_{\text {off }} \sinh \left(\frac{i}{i_{\text {off }}}\right) \exp \left[-\exp \left(\frac{w-a_{\text {off }}}{w_{c}}-\frac{|i|}{b}\right)-\frac{w}{w_{c}}\right]
$$

Doped:


## Experimental evidence for Front model

## 1. Direct imaging

Shows dynamics of front explicitly
2. HP group analyzed scaling of I-V with lateral width of contact: front/channel in TiO: $x \sim 50-100 \mathrm{~nm}$
3. In devices of decreasing size: do front and channel pictures converge? Device lateral size can get in the same range of $\sim 30-50 \mathrm{~nm}$


However, in small devices the channels can get smaller as well, only few nm, still distinct from front of size, comparable to system size

## HP Modeling: Strukov et al.

Vacancies and electrons couple only through Coulomb interaction:

1. Both of them are sources of the potential $\phi$

$$
\begin{aligned}
& -\varepsilon \varepsilon_{0} \Delta \varphi(x) \\
& \quad=e\left(p(x)-n(x)+f_{\mathrm{D}}(x) N_{\mathrm{D}}(x)-f_{\mathrm{A}}(x) N_{\mathrm{A}}\right)
\end{aligned}
$$

2. The dynamics of both of them is driven by $\phi$

$$
\begin{aligned}
& J_{\mathrm{ION}}(x)=-e D_{\mathrm{i}} \nabla N_{\mathrm{D}}(x)-e N_{\mathrm{D}}(x) u_{\mathrm{i}} \nabla \varphi(x) \\
& \nabla \cdot\left(-e n(x) \mu_{\mathrm{n}} \nabla \varphi_{\mathrm{n}}(x)\right)=0
\end{aligned}
$$


3. Electron mobility is not effected by vacancies

## HP Modeling: Strukov et al.

HP simulations

4. Simulation: $w(O N)=0$

Self arrest: NO

HP measurement of ON/OFF width


Expt: $w(O N)=1.4 \mathrm{~nm}$ YES

## So, we decided to check out the vacancies


by checking in the Bates Motel

## What could possibly go wrong?



## Our Simulations



Motivated by $\mathrm{TiO}_{2} / \mathrm{TiO}_{(2-x)}$ layered binary oxide structures

Pre-formed system, no additional formation is needed: the front separating high and low vacancy density regions is prepared

Can be equally appropriate for a formed channel

Thickness of insulating $\mathrm{TiO}_{2}$ layer is about 2 nm or less
(1) calculate the full energy of the electron system, driven by an applied voltage
(2) move the electrons with Monte-Carlo dynamics until a steady current state is established (1-100 million MC steps)
(3) move the vacancies according to a MonteCarlo dynamics using the electron configuration related to steady current
(4) recalculate the electronic current
(5) repeat the vacancy \& electron updates until both reach steady state, then record the current, corresponding to the applied voltage
(6) change the voltage incrementally and repeat steps (1)-(5)

## Energetics

## Electron energy

$$
\begin{gathered}
E_{i}=C^{s}\left(n_{i}-N_{\mathrm{avg}}\right)^{2}+\frac{C^{c}}{2} \sum_{k \neq i} \frac{\left(n_{k}-N_{\mathrm{avg}}\right)\left(n_{i}-N_{\mathrm{avg}}\right)}{d_{i k}}+C^{p} V_{i} n_{i}+C^{\mathrm{d}} L_{\mathrm{i}} n_{\mathrm{i}} \\
\begin{array}{c}
\text { grain charging } \\
\text { energy }
\end{array} \\
\begin{array}{l}
\text { Coulomb } \\
\text { interaction }
\end{array}
\end{gathered} \begin{aligned}
& \text { potential due to } \\
& \text { external voltage }
\end{aligned} \begin{gathered}
\text { disordered } \\
\text { grain energy }
\end{gathered}
$$

Electrodes
Electron reservoirs, separated from bulk with a work function W

## Dynamics

## Electron dynamics

$\delta_{j}<\delta_{0}$ : tunneling, in boundary layer
$\mathrm{p}(i \rightarrow j)=\mathrm{p}_{0} \exp \left(-\left(\mathrm{V}_{0}+\Delta \mathrm{E}_{i j}-\right.\right.$ const. $\left.\left.\delta_{j}\right)\right) \Theta\left(-\Delta \mathrm{E}_{i j}\right)$
Tunneling barrier lowered by energy gradient and vacancy density
$\delta_{j}>\delta_{0}$ : metallic, in bulk
$p(i \rightarrow j)=p_{0} \Theta\left(-\Delta \mathrm{E}_{i j}\right)$
Vacancy dynamics

$$
\delta \rho(i \rightarrow j)=\mu \exp \left(-\left(V_{00}-\Delta V_{i j}\right) / E_{0}\right)
$$

Mott-Guerney type

## Results: No mobile vacancies

Onset/switching could have been observed without vacancy dynamics:
(a) Anderson localization
(b) Interacting localization (Coulomb Glass)
(c) Some kind of depinning
(d) Mott transition

Instead: no hysteresis, no switching



Mobile vacancies are essential for switching phenomena

## Results: Mobile vacancies



FIG. 6. IV curves with varying disorders


WO3/PT cell

## Results: Parameter space exploration




Different types of I-V curves
$\mathrm{R}_{\text {on }} / \mathrm{R}_{\text {off }}$ ratio as a function of parameters

## Switching Mechanism

Applied Voltage: 0, Current: -0

depletion layer
4. This reduces field, arrests further front motion


## Comparison to other work

## Rozenberg, Sanchez, Levy et al



## Our Model - Next Generation



- 3000 atoms
- Random energies
- Coulomb interaction (100,000 grid point)
- Electrons jump by master eq.


## Summary

1. Reviewed some existing simulations, their applicability and limits
2. We studied the "coupled mobile electrons-mobile vacancies" model with explicit electron dynamics, having long range interactions and mobile vacancies
3. Switching requires mobility of vacancies
4. With mobile vacancies model reproduces experiments promisingly
5. Switching is driven by hole pile-up, flushed out with ON switching
6. Switching is - self-arresting

- sharp onset without assumption of double exponential w(i)
- boundary layer is self-defined


## Broad Distribution of Switching Parameters



## Memristors everywhere


(a) $\mathrm{Au} / \mathrm{Ti} / \mathrm{SrZr} 0.998 \mathrm{Cr} 0.002 \mathrm{O} 3 / \mathrm{SrRuO} 3$; (b) $\mathrm{Ag} / \mathrm{CeO} 2 / \mathrm{La} 0.67 \mathrm{Ca} 0.33 \mathrm{MnO} 3$;
(c) $\mathrm{Ag} / \mathrm{Bi} 2 \mathrm{Sr} 2 \mathrm{CaCu} 2 \mathrm{O} 8+y$ heterojunction; (d) $\mathrm{Pt} / \mathrm{NiO} / \mathrm{Pt}$; (e) $\mathrm{Al} / " R o s e ~ B e n g a l " / I T O ;$
(f) Al/DDQ/ITO; (g) Au/porus Si/p-type Si; (h) Double barrier AlAs/GaAs heterostructure.

## Hysteretic/switching resistors: Bednorz strikes gold again

## Reproducible switching effect in thin oxide films for memory applications

A. Beck, J. G. Bednorz, Ch. Gerber, C. Rossel, ${ }^{\text {a }}$ ) and D. Widmer

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$\mathrm{SrTiO}_{3}, \mathrm{SrZrO}_{3}$


