

# Aging and Coarsening in Dislocation Glasses

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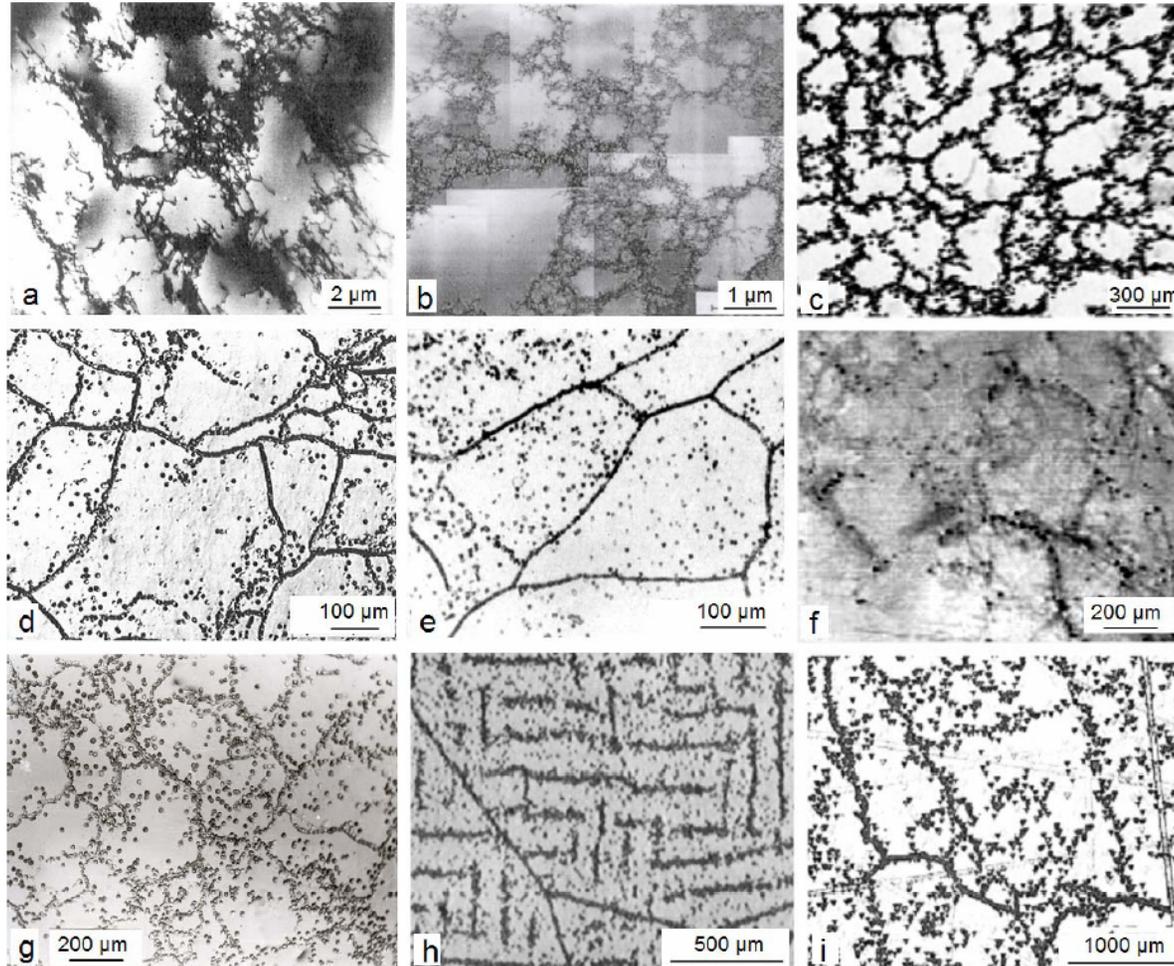
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# Aging and Coarsening in Dislocation Glasses

- I. Pattern formation
- II. Concepts of non-equilibrium phenomena
- III. Numerical simulations of dislocations
- IV. Glide, no climb: Aging, Freezing
- V. Climb: Domain formation, Coarsening
- VI. Wall formation: mean field, beyond mean field
- VII. Role of Anchors

# I. Pattern Formation



**Fig. 1** Dislocation patterns formed in various crystals under differing stress conditions. (a) Mo 12% deformed at 493 K [1]; (b) Cu-Mn crystal deformed at 68.2 MPa [2]; (c) GaAs crystal grown by VCz (author's image); (d) CdTe crystal grown by VB (authors image); (e) PbTe crystal grown by VB [33]; (f) SiC crystal grown by sublimation (courtesy of D. Siche from IKZ Berlin); (g) Cd<sub>0.96</sub>Zn<sub>0.04</sub>Te crystal grown by VB (author's image); (h) NaCl crystal with labyrinth structure deformed by 150 MPa at  $T/T_m = 0.75$  [3], (i) CaF<sub>2</sub> crystal grown by VB [4].

# I. Pattern Formation

Pattern formation in dislocation systems is ubiquitous



Equilibrium statistical physics: low  $T$  phase is dilute gas of dislocations with density vanishing as  $T \rightarrow 0$

Pattern formation is a far from equilibrium phenomenon

# I. Pattern Formation

## Early approaches

1. Rate equations (Kocks)
2. Mobile and immobile dislocations, transitions between (Kubin)
3. Intersecting dislocation loops (Friedel, Kubin)
4. Reverse diffusion (Holt)
5. Forward Diffusion + Two types of dislocations (Walgraef-Aifantis)
6. Statistical ensembles (Ananthakrishna)

## II. Concepts of Non-equilibrium Dynamics

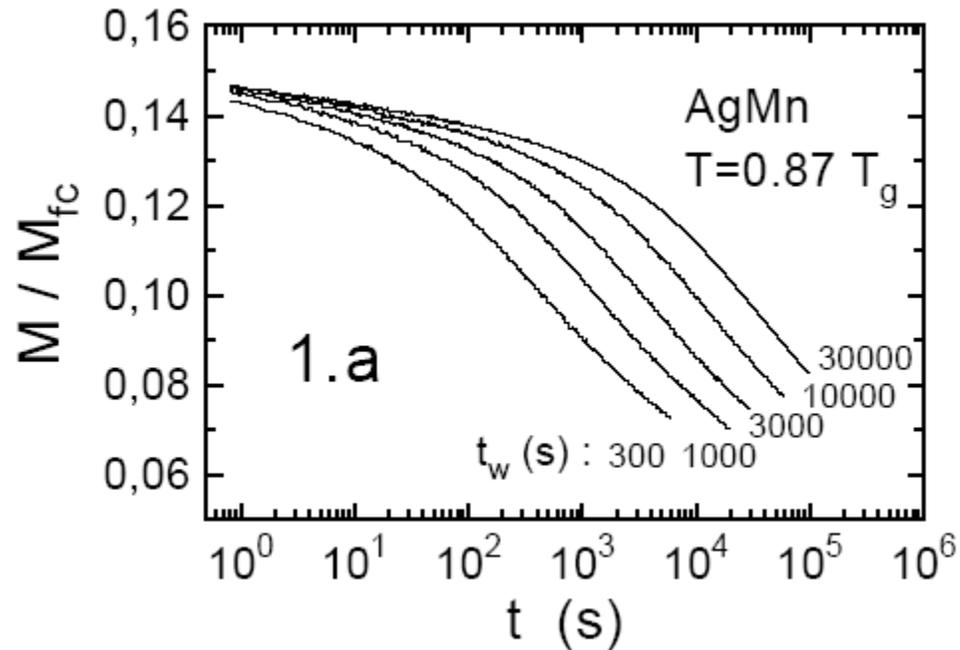
**Start** system far out of equilibrium, let it relax

**Three hallmarks of glassiness**

- 1. Freezing of dynamics:** time scale slows down by many orders of magnitude
- 2. Aging:** Response depends on a waiting time
- 3. Coarsening:** Domains form, grow in time

(This work: only annealing, no shear)

## II. Aging

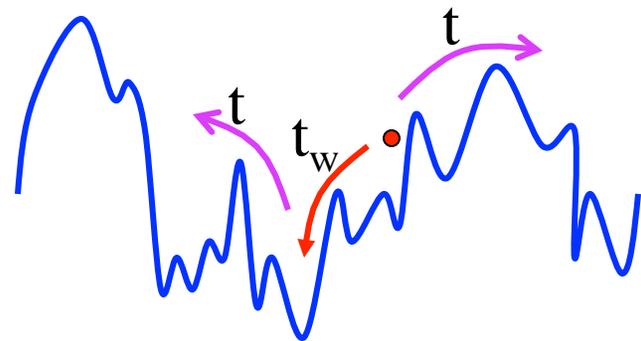


**Response depends on the waiting time:**

$$C(t, t_w) \neq C(t - t_w)$$

Spin Glass: Ag+2%Mn

1. Field cool
2. Wait for  $t_w$
3. Measure relaxation



## II. Aging

In mean field theory,  $C(t, t_w)$  can assume scaling forms:

1. Full/simple aging:  $C(t_w, t+t_w) = C_{eq}(t) C_{aging}(t/t_w)$

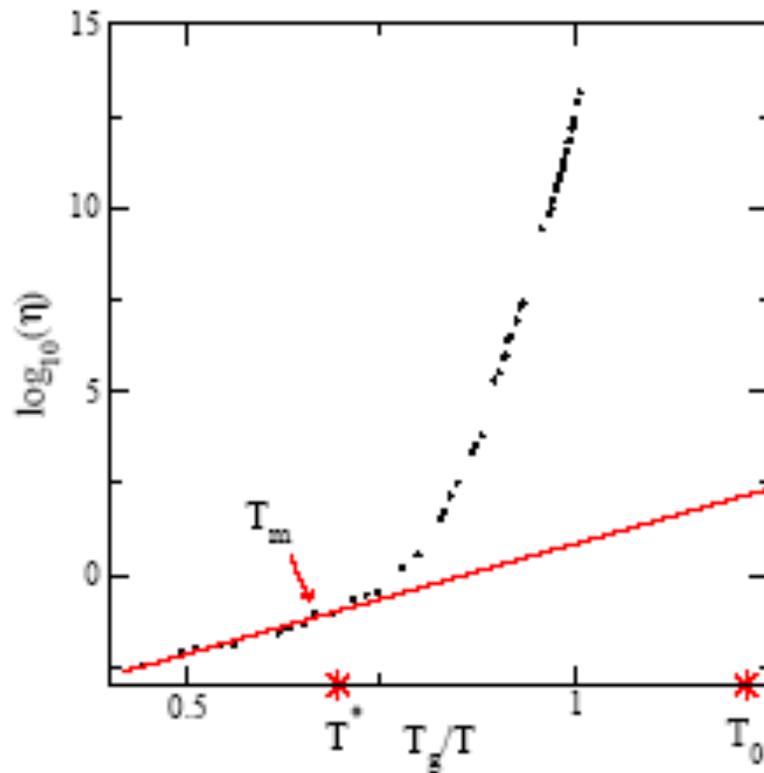
2. Super/sub-aging:  $C(t_w, t+t_w) = C_{eq}(t) C_{aging}(t/t_w^\mu)$

$\mu > 1$  (superaging) and  $\mu < 1$  (subaging) have been observed experimentally and numerically

3. Activated aging:  $C_{aging}(h(t+t_w)/h(t)) = C_{aging}(\ln(t+t_w)/\ln(t))$

Worked very well for the 3D Heisenberg spin glass.

## II. Freezing of Dynamics



Super Arrhenius law for  
viscosity:

1.  $T_G$  finite,  $\tau \sim \exp[1/(T-T_G)]$

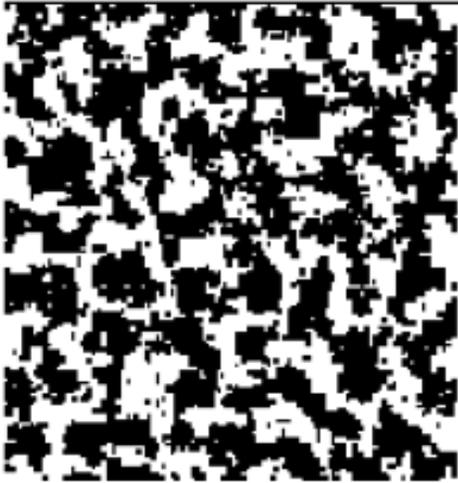
Vogel-Fulcher

2.  $T_G$  zero: avoided criticality

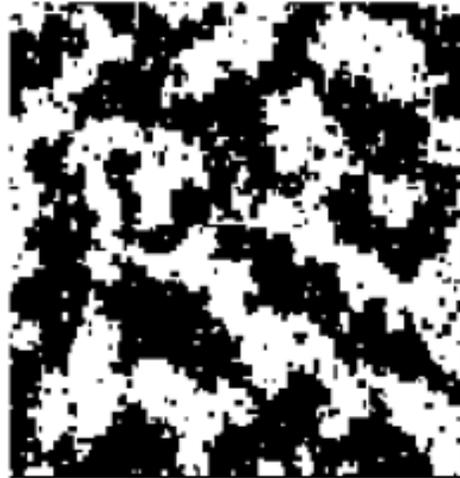
Kivelson

In practice: hard to distinguish,  
 $\tau$  becomes immeasurably  
large at finite  $T$

## II. Coarsening



$t=10^3$  MC steps



$t=10^5$  MC steps

3D ordered Ising model,  
Anneal to  $T < T_c$

1. Domains form
2. Domains grow

## II. Coarsening: Egyptian vases



Domains with increasing sizes  
form on a time scale of thousands of years

## II. Coarsening: Ordered, Disordered Systems

### 1. Ordered systems: $L \sim t^{1/z}$ , $z=2$

Theory: infinite D, mode coupling: hard to identify length

Recently, Landau theory: growing length scales studied

(Chamon et al, 2002)

Results can depend on dynamics: Kawasaki:  $z=3$

### 2. Disordered systems: $L \sim (T \log t)^{1/\psi}$ “ $z=0$ ”

Energy barriers scale as  $E \sim L^\psi$

Time to overcome barriers by activated dynamics:

$$t \sim \exp(E/kT) \sim \exp(L^\psi/kT)$$

Non-frustrated randomness (RFIM): Yes

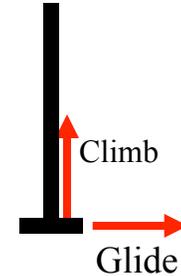
Frustrated randomness (EA): less clear

# III. Simulations: Aristotelian Dynamics

Kroner continuum formulation:

$$D\Delta^2 \chi = (b_x \partial_y - b_y \partial_x) \rho, \quad \sigma_{ij} = \frac{\partial^2 \chi}{\partial x_i \partial x_j}, \quad f_{PK}^r = \sigma b \hat{z}$$

$$\vec{v} = B_g \vec{n}_g \tau_g^{PK}(r) + B_c \vec{n}_c \tau_c^{PK}(r)$$

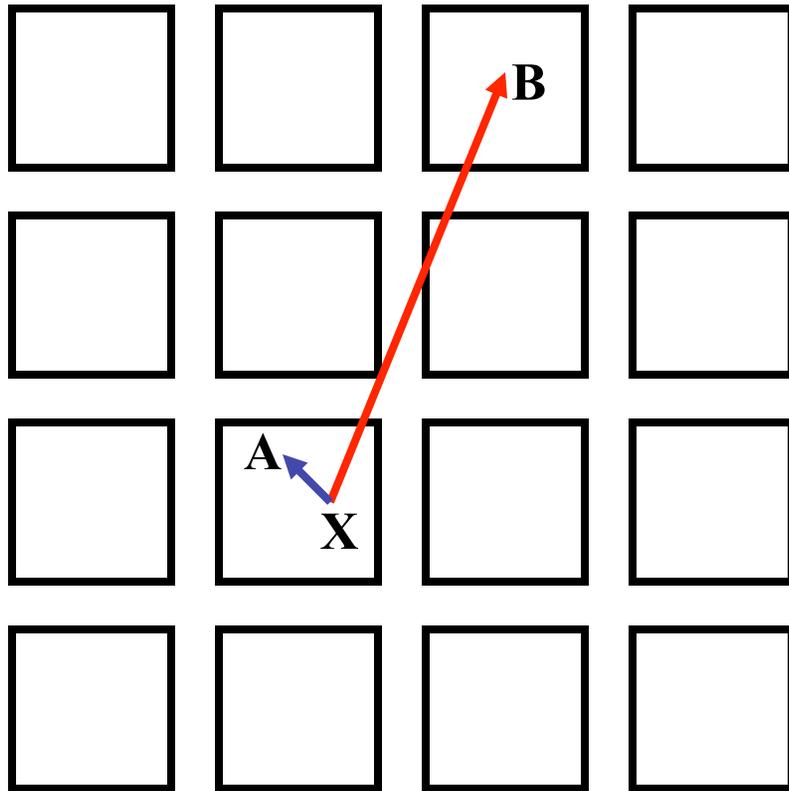


1. Overdamped (Aristotelian) dynamics
2.  $\tau_{g/c}$  is the glide/climb component of the stress-related Peach-Kohler force
3. Dislocation interaction is in-plane dipole-dipole type (“vector Coulomb gas”)
4. No external disorder

## Features

1. Glide and climb (mobility:  $B_g, B_c$ )
2. Annihilation
3. Thermal force
4. Advanced acceleration technique
5. Rotation

# III. Fast Fourier Multipole Expansion



1. Divide simulation space into boxes:  
40,000 dislocations, 60,000 boxes
2. Calculate intra box interactions  
AX in real space
3. Calculate interbox interactions BX  
by calculating stress by Fast  
Fourier transformation
4. Move dislocations by eq. of motion
5. Repeat from 2

## III. Simulation Features

Polarized - Non-polarized

1 glide axis - 3 glide axes

Climb - No Climb

Annihilation - No annihilation

$T=0$  -  $T>0$

Rotation - No rotation

## IV. Simulation Results: No Climb

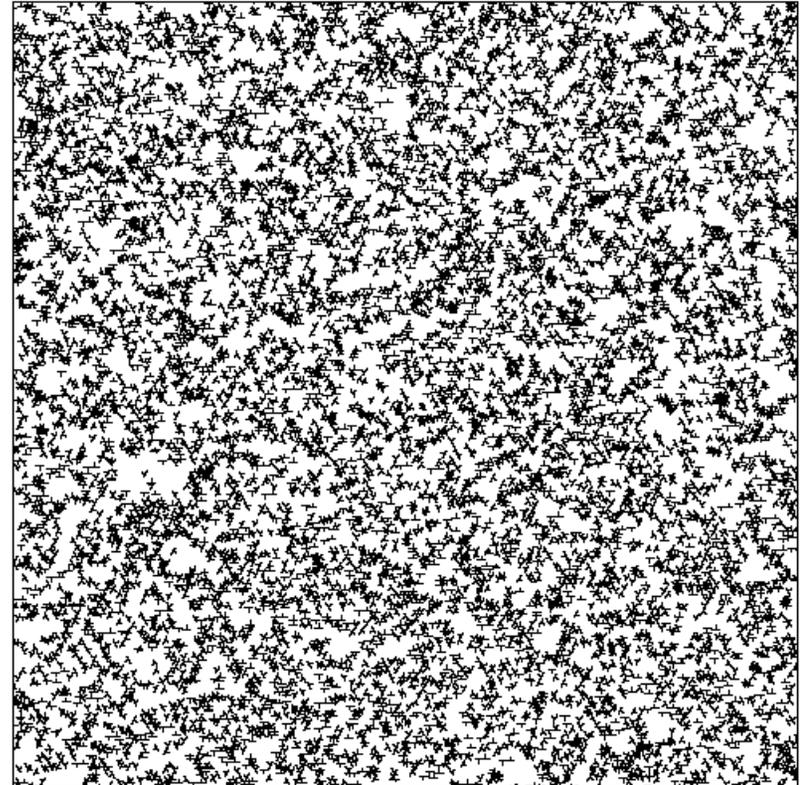
3 Glide axes

Non-polarized

Glide only, no Climb

No annihilation

**Limited structure  
formation**

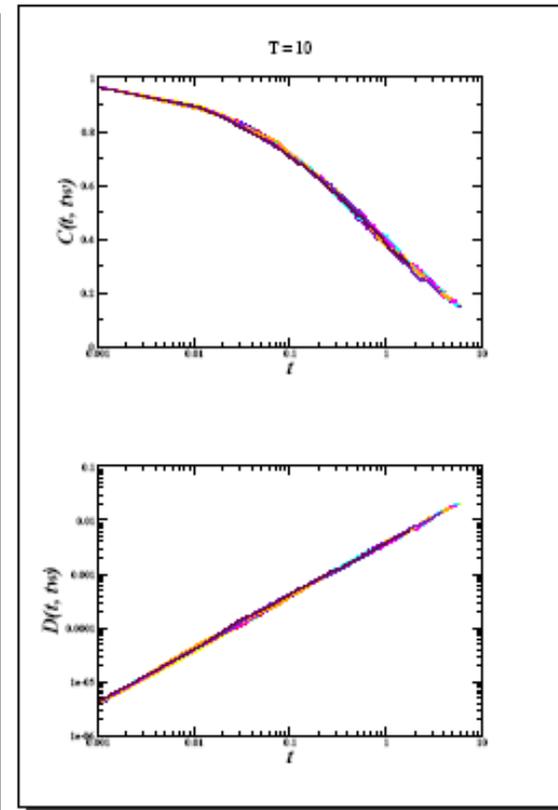
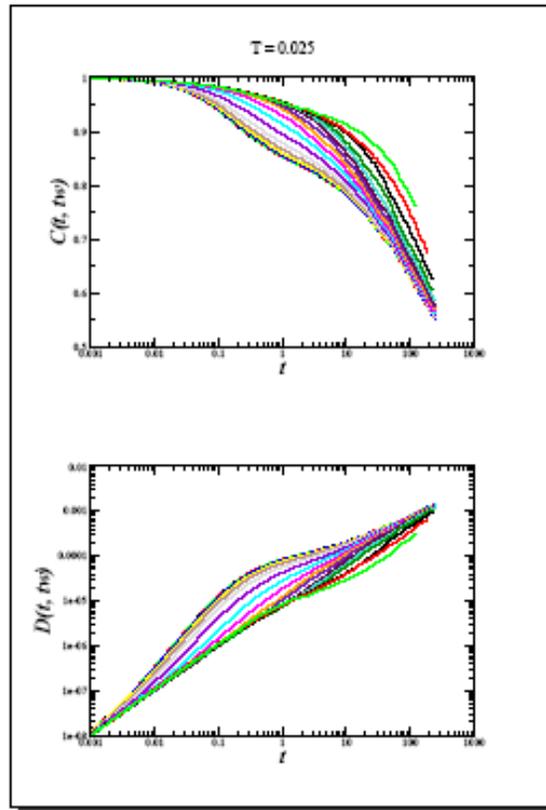


# IV. No Climb: Aging

T=0.025

T=10

Self-overlap



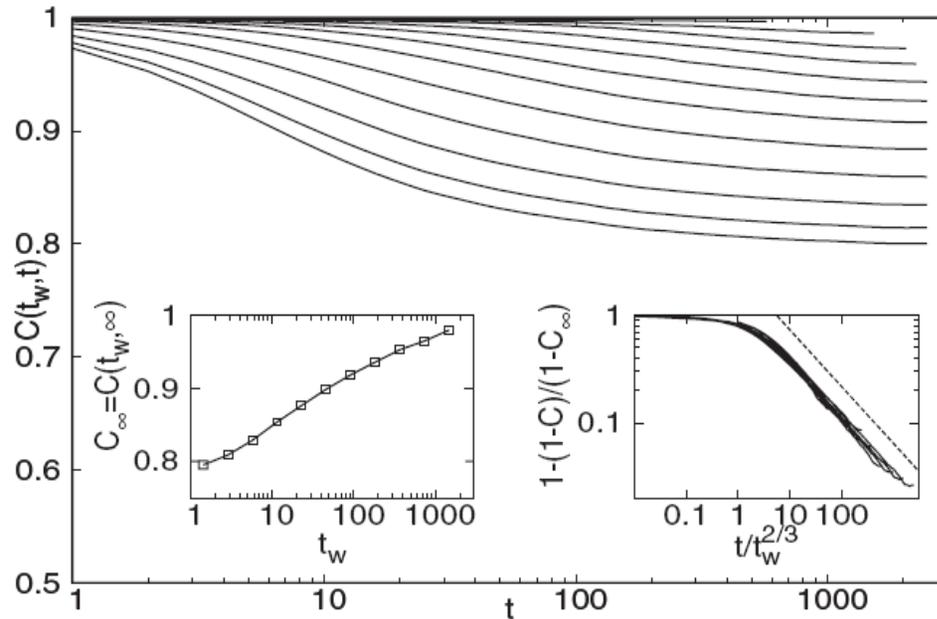
Effective diffusion

$$C(t, t_w) = \frac{1}{N_d} \sum_{i=1}^{N_d} \exp[-const |\vec{r}_i(t+t_w) - \vec{r}_i(t_w)|] \quad D(t, t_w) = \frac{1}{N_d} \sum_{i=1}^{N_d} |\vec{r}_i(t+t_w) - \vec{r}_i(t_w)|^2$$

**At low temperatures system falls out of equilibrium**

**Measured quantities start depending on waiting time: Aging**

## IV. No Climb: (Near) Textbook Aging



$$C(t_w, t) = \frac{1}{N_d} \sum_{i=1}^{N_d} \exp[-|\mathbf{r}_i(t + t_w) - \mathbf{r}_i(t_w)|/r_0]$$

$$C(t, t_w) = C_{\text{eq}}(t) C(t/t_w^\mu)$$

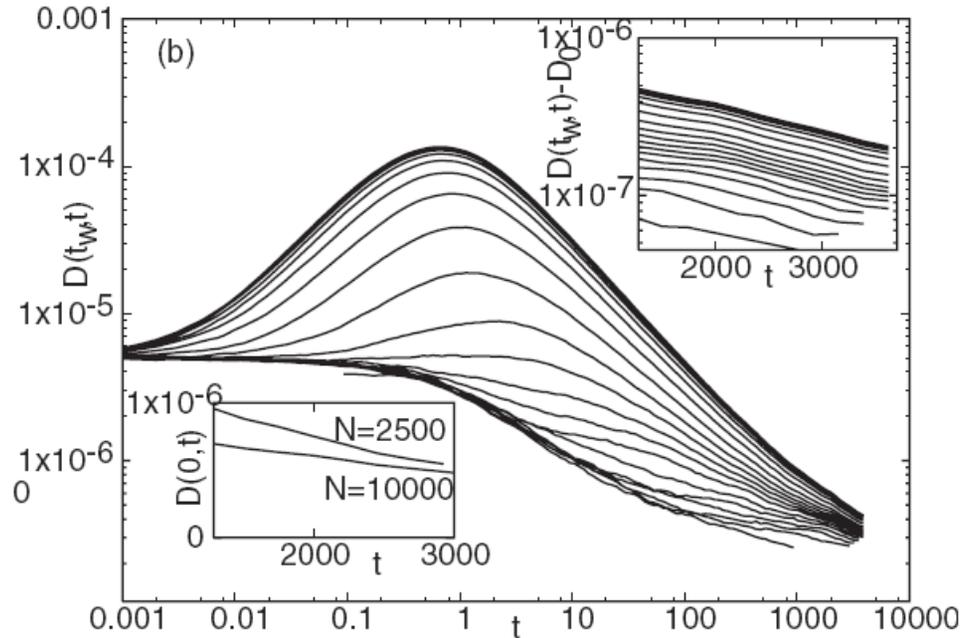
$$C_{\text{eq}}(t) \sim t^{-\beta}$$

$$\mu = 0.66, \beta = 0.54$$

Analogous to spin glasses,  
since the location of the axes  
is a quenched randomness

$\mu$  is close to  $\beta$

## IV. No Climb: Freezing



$$D(t, t_w) \sim D(t_w) t^{-\gamma} \quad T > 0$$

$$\gamma = 0.8$$

Diffusion constant goes to 0:

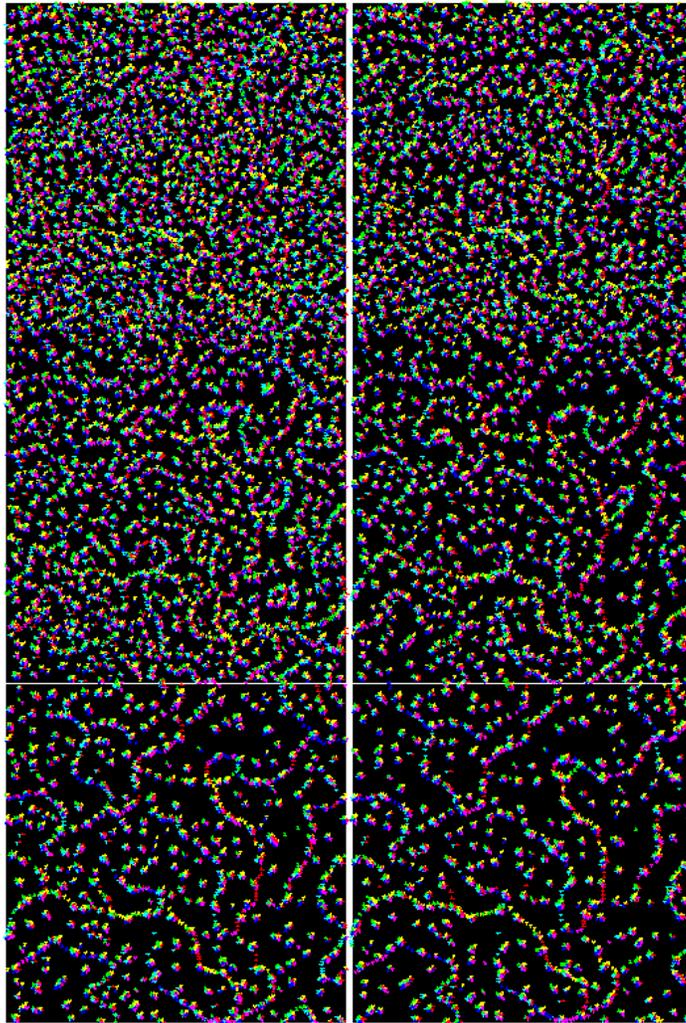
**Freezing of dynamics**

Aging and Freezing are  
evidence for:

**Dislocation Glass**

$$D(t_w, t) = \frac{1}{N_d t} \sum_{i=1}^{N_d} |\mathbf{r}_i(t + t_w) - \mathbf{r}_i(t_w)|^2$$

# V. Climb+Annihilation: Wall/Domain Formation



$$B_c/B_g=0.1$$

**Glide, 3 slip axes, non-polarized:  
add climb, annihilation**

- **Glide only:**

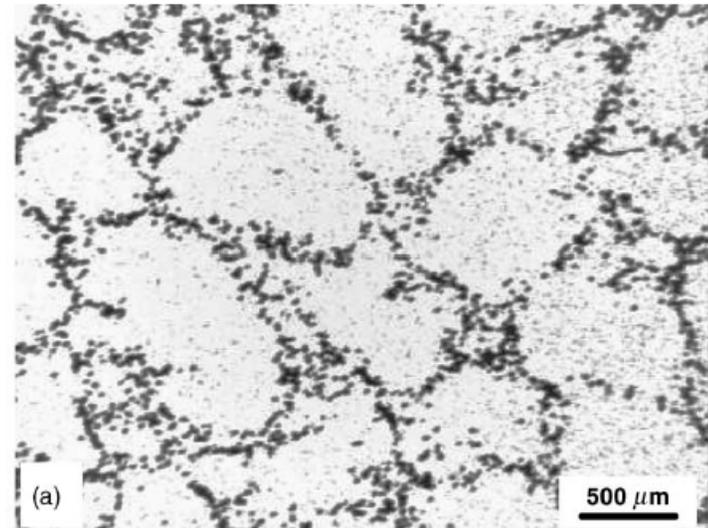
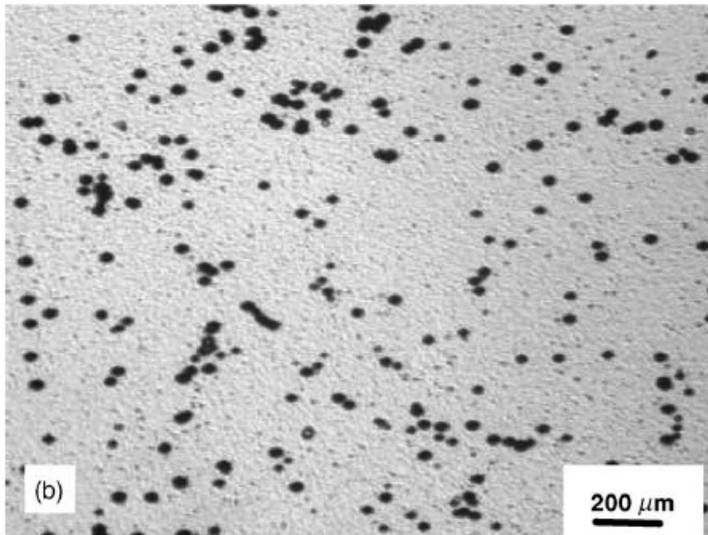
10% of dislocations form walls,  
90% remains in dipoles,  
which cannot annihilate

- **Glide+climb, annihilation:**

Dislocations outside walls can  
annihilate, only walls remain

**Domain formation induced  
by climb**

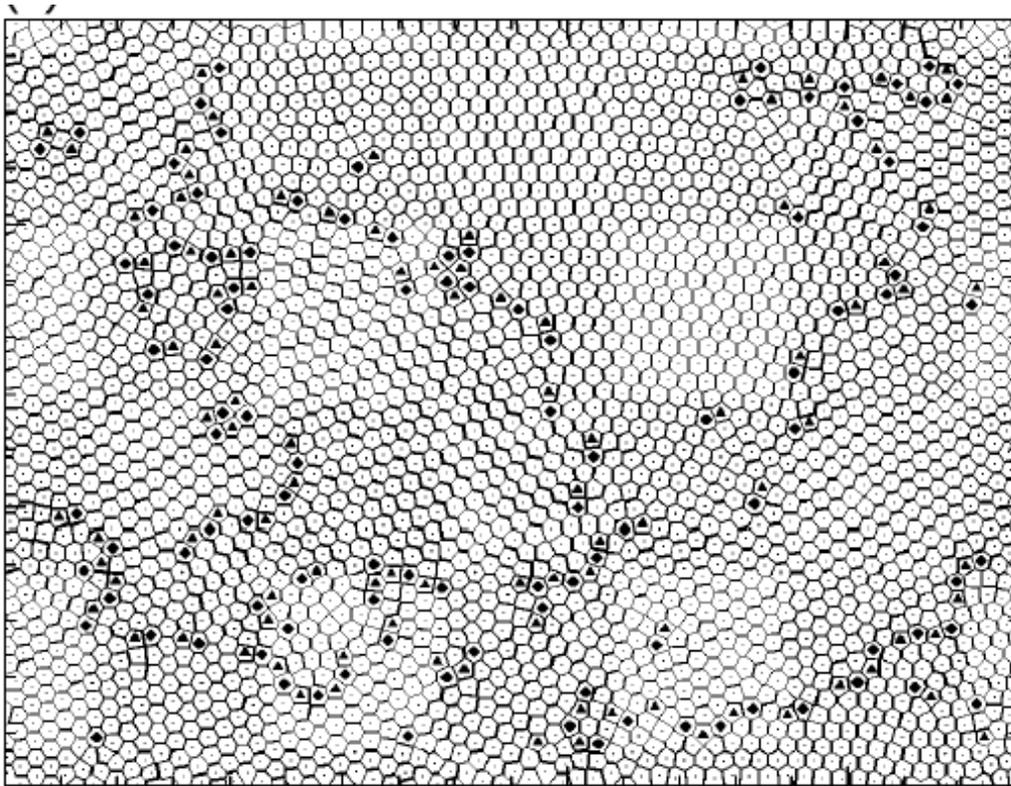
# V. Climb: Experiment: Climb Induces Domain Structures in GaAs



Rudolph et al. (2005)

Materials Science and Engineering A 400–401 (2005) 170–174

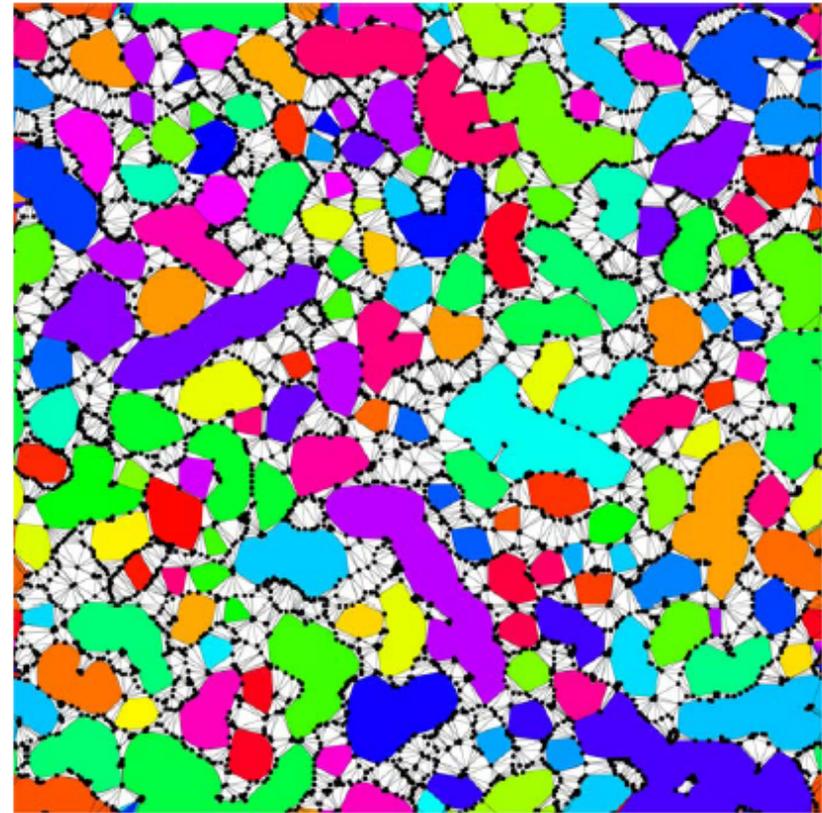
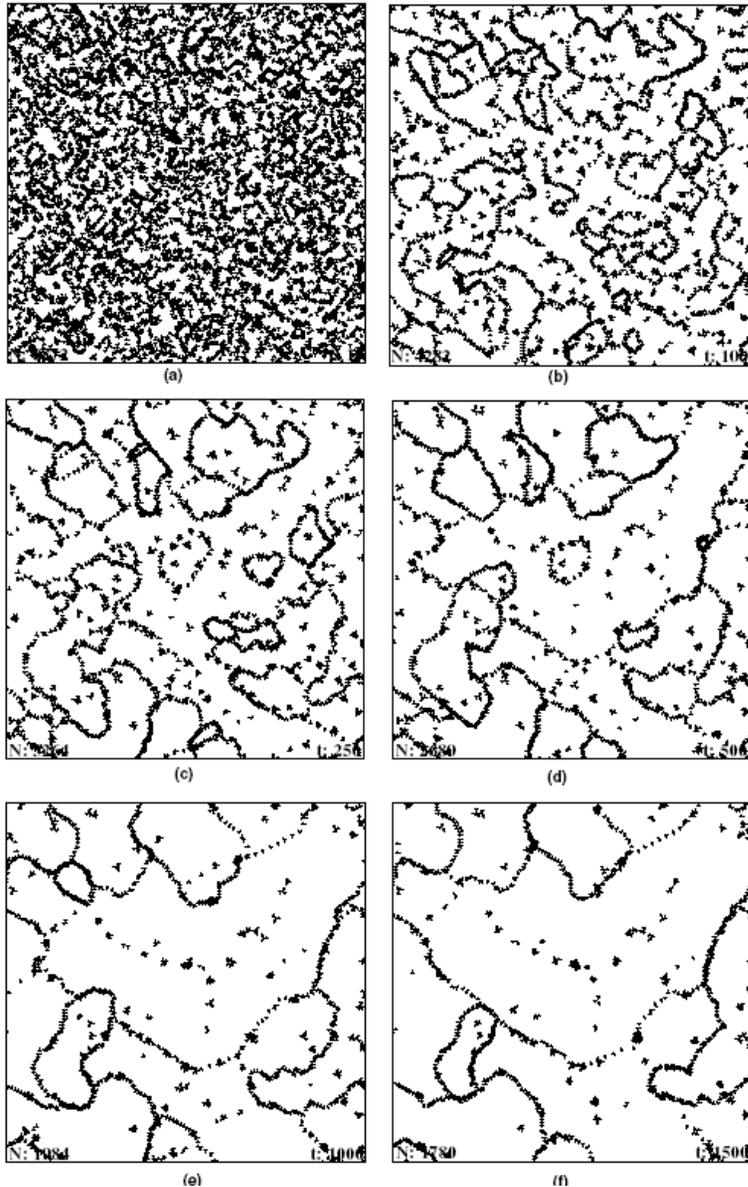
# V. Climb: Experiment: Domain Formation in Dusty Plasmas



Charged particles settle  
Climb is present  
Domain formation

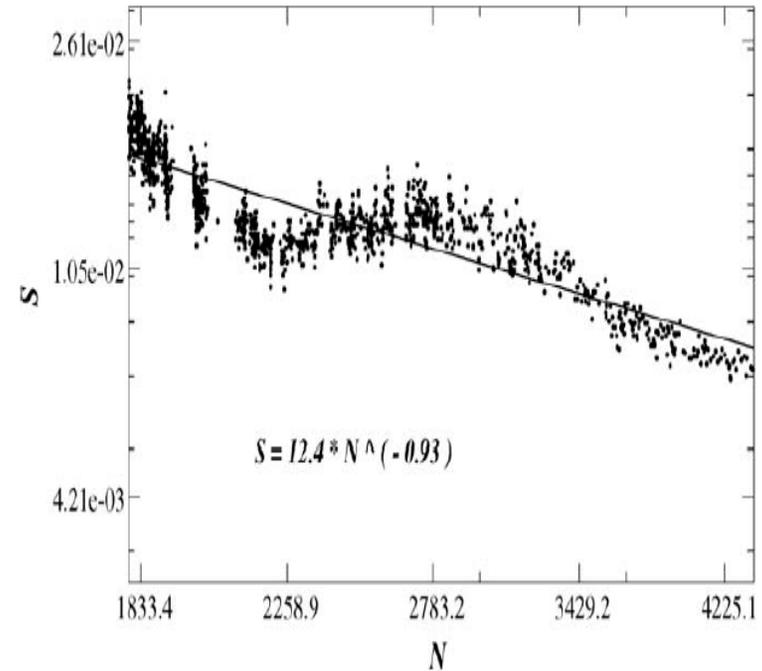
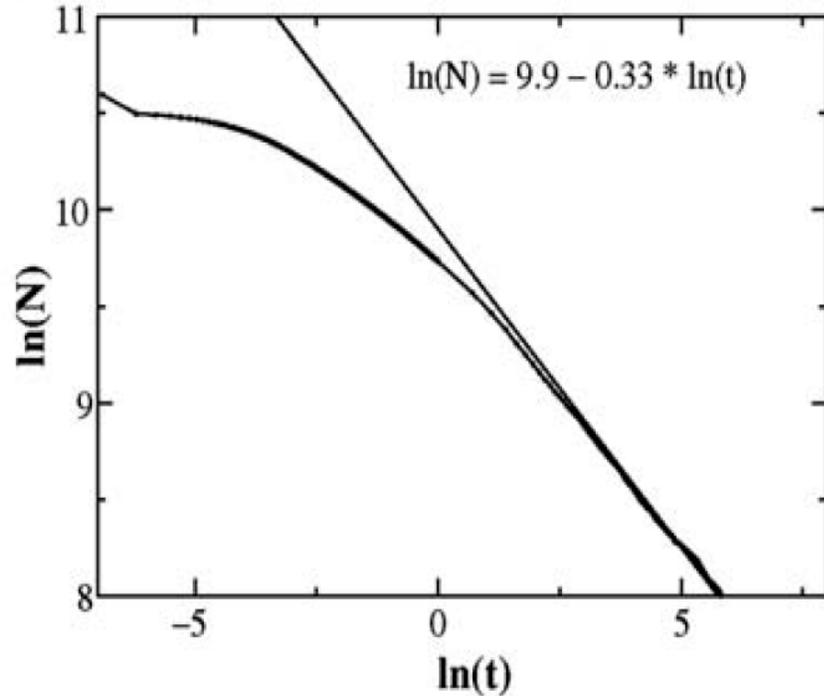
Quinn, Goree, 2001

# V. Climb: Coarsening



Domain size grows with time  
Number of dislocations decreases

# V. Climb: Coarsening: z, Holt relation



Number of dislocations:  $N \sim t^{0.33}$

Ave. distance between dislocations  $L \sim 1/N^{1/2}$

$$L \sim t^{1/z}$$

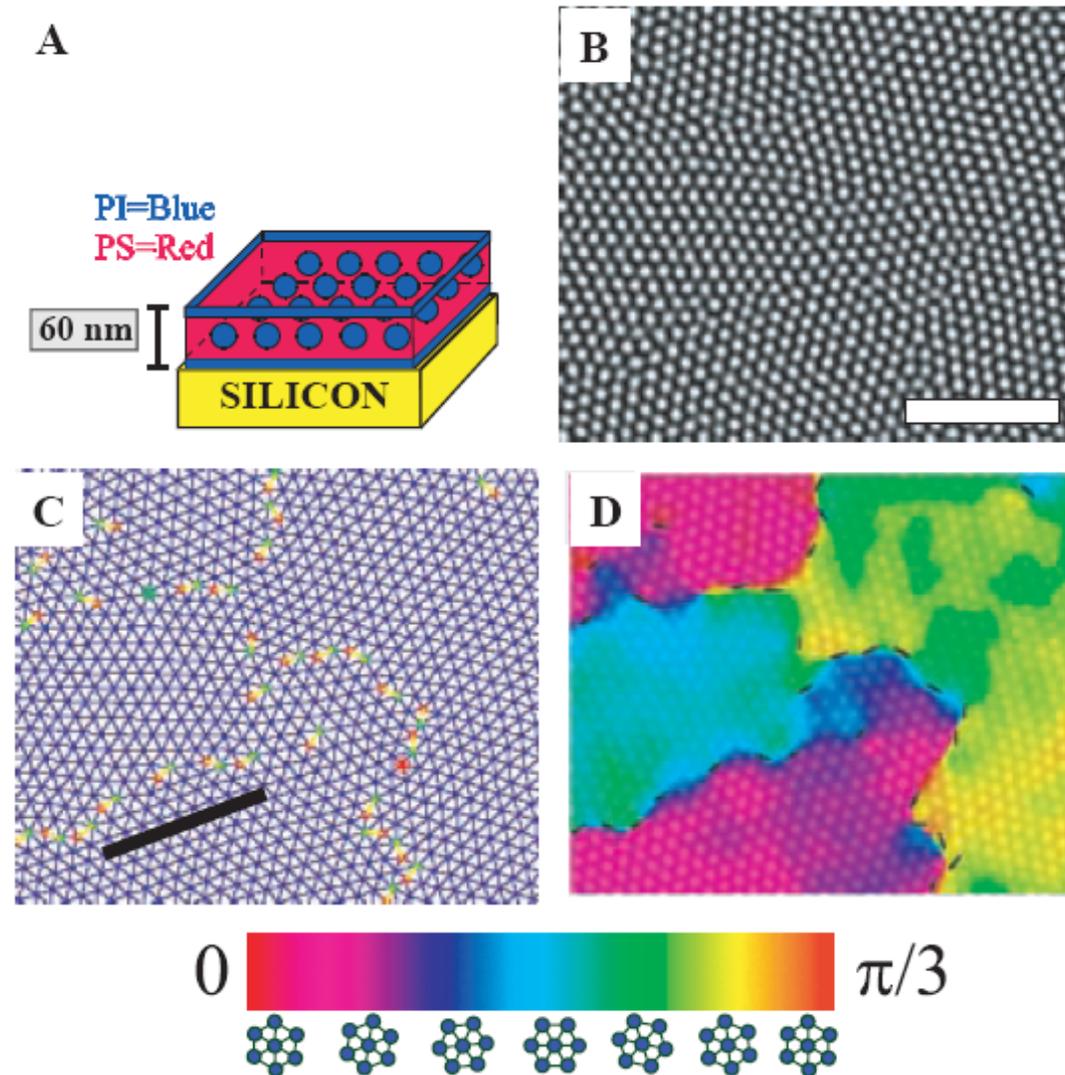
$$1/z \sim 0.17$$

**Holt relation:** cell area  $S$

$$S \sim N^{-1}$$

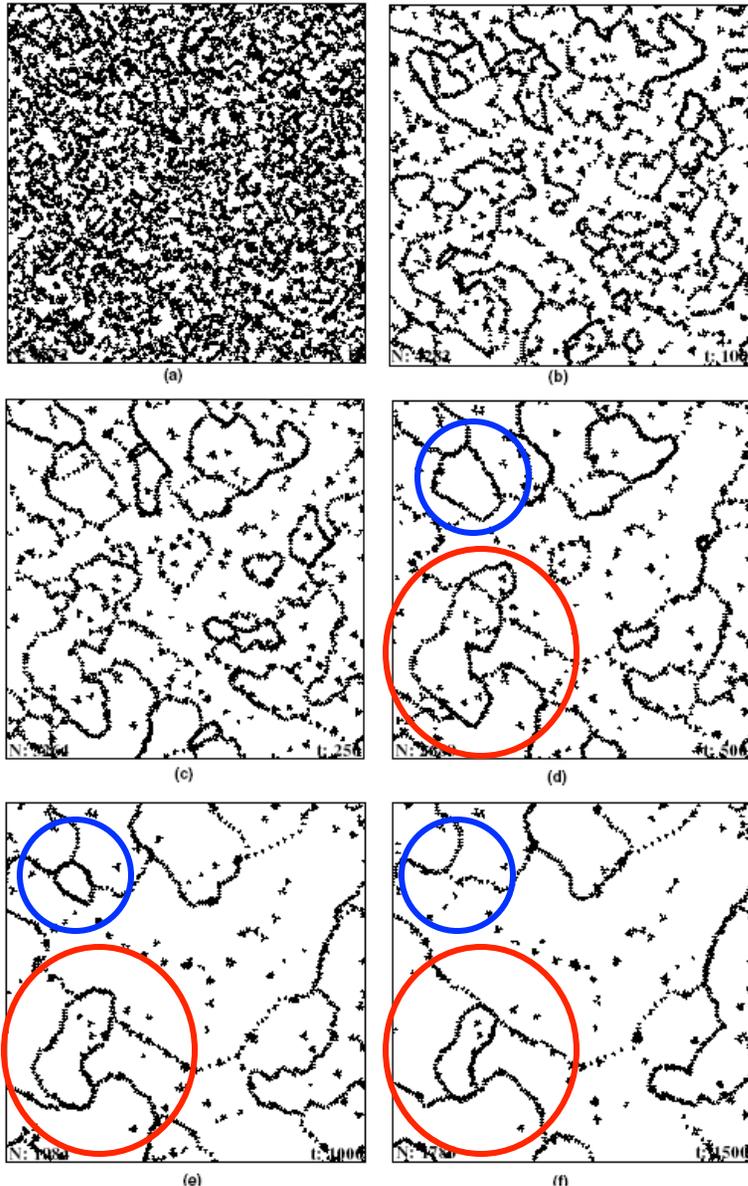
We measured  $S$  independently by a domain identifying search

# V. Coarsening in Di-block copolymers



Chaikin, Huse, 2004  
previous talk

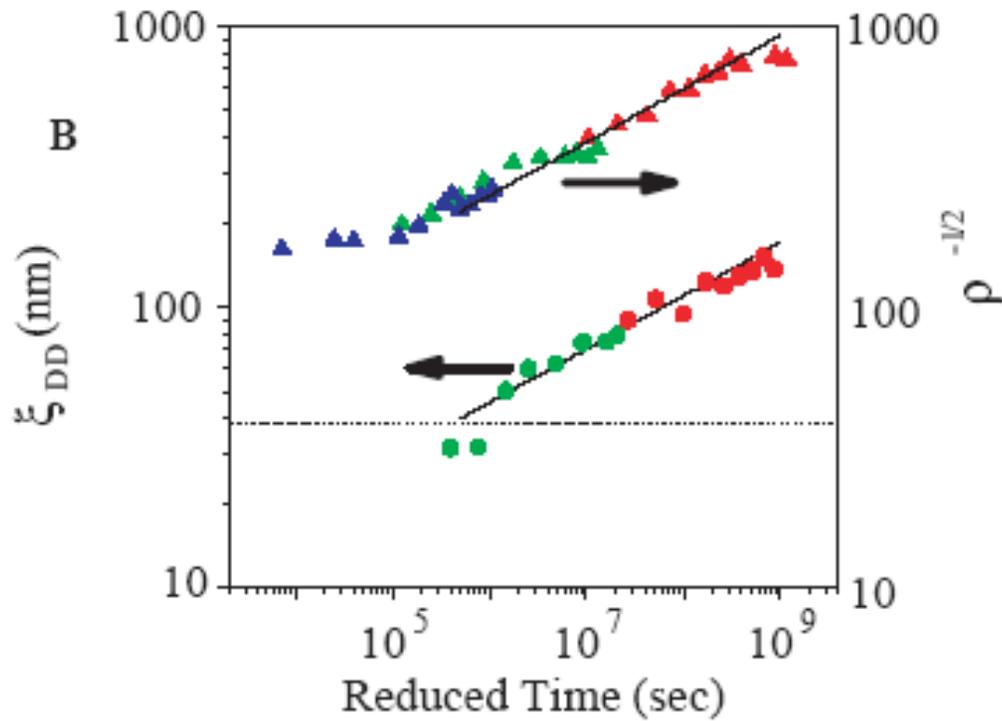
# V. Coarsening by Domain Absorption



Coarsening happens by smaller domains getting absorbed at the boundaries of bigger domains.

Connection to polymers

# V. Coarsening Exponent: $1/z=0.19$



**Remarkable agreement  
with our result of  $1/z=0.17$**

# V. Summary of Part I.

## 1. Glide only model

- Aging: sub-aging scaling with waiting time
- Freezing: effective diffusion constant goes to zero
- Evidence for **Dislocation Glass**
- Limited domain formation

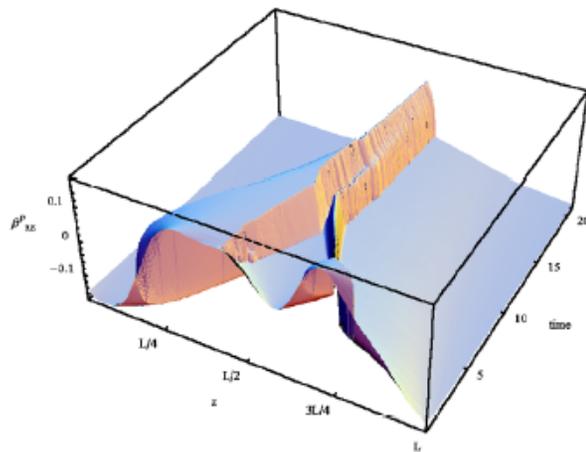
## 2. Glide + climb model

- Domain formation
- Coarsening with exponent related to experiment

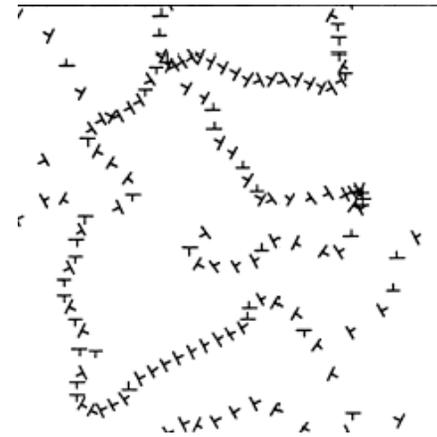
Proceed to understanding domain wall formation in detail

# VI. Understanding Wall Formation

Sethna-Linkumnerd  
(2006)

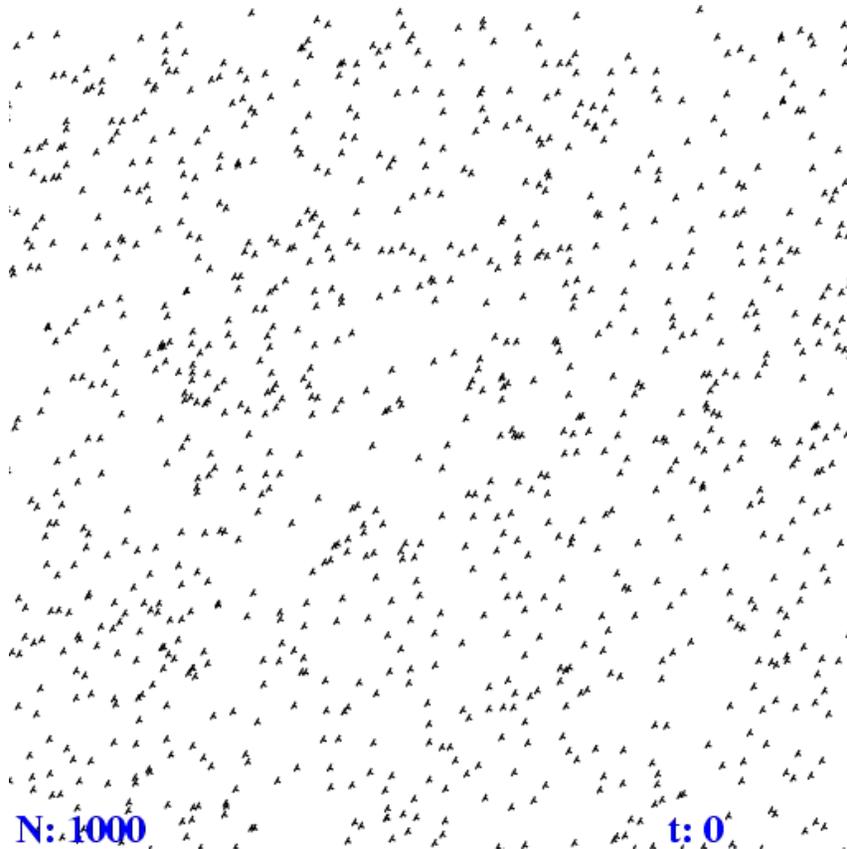


Argaman (2001)

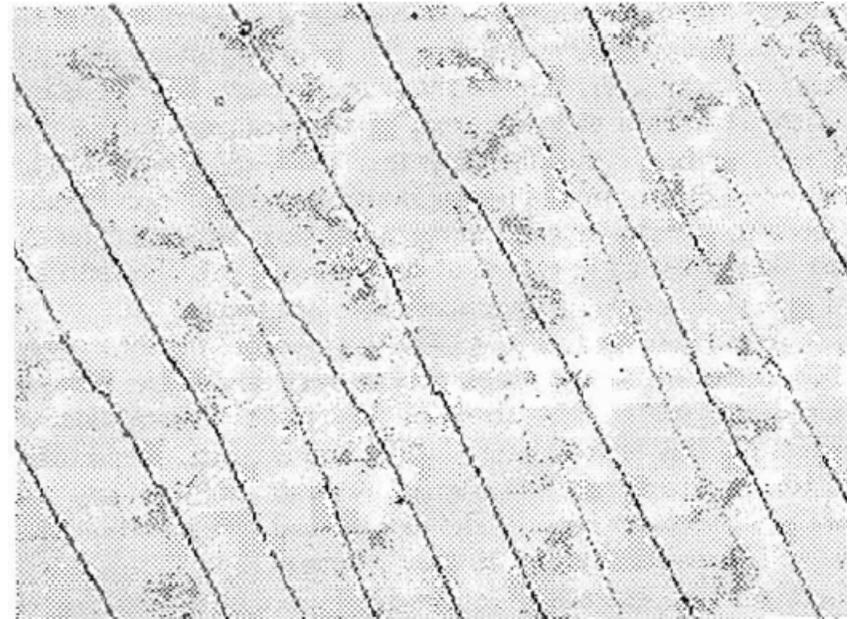


also, Barts-Carlson (1995)

## VI. Glide only, Polarized: Wall Formation

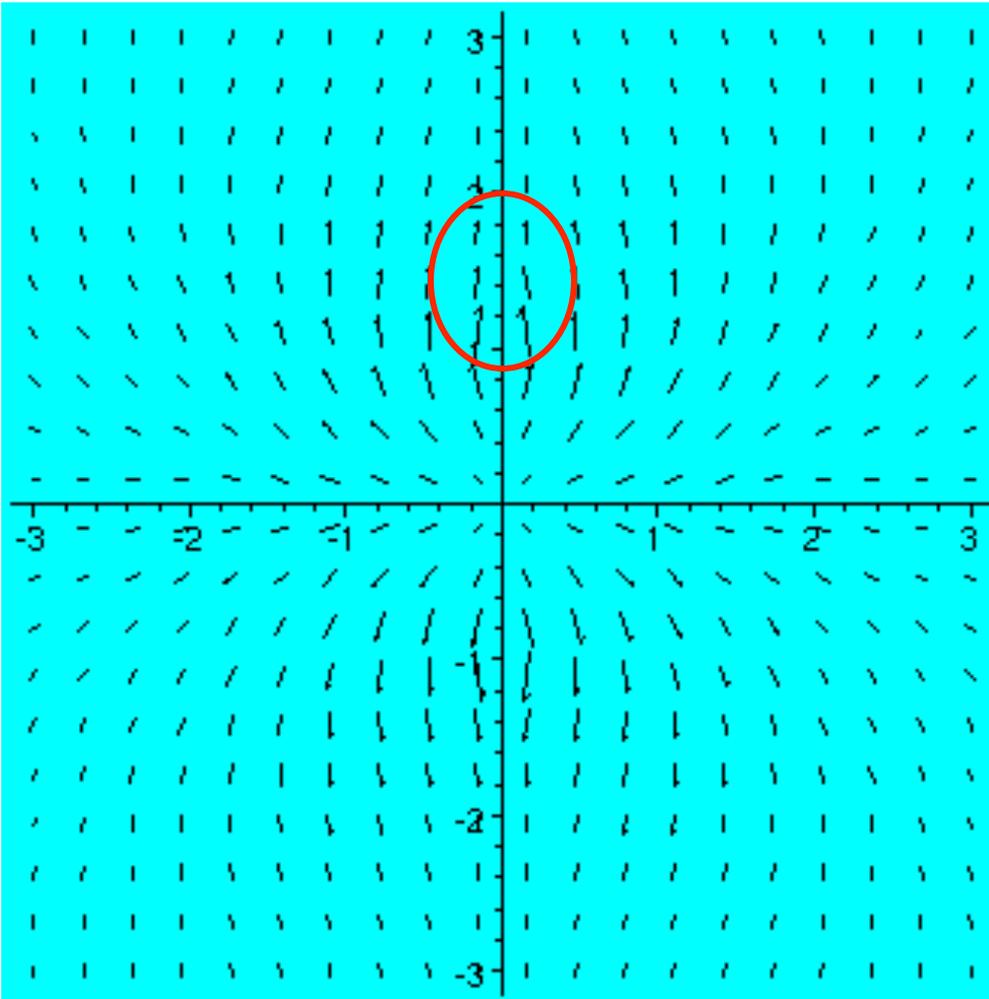


- Dislocations glide along 1 axis
- Walls are energetically favorable...



Fe-Si, Hibbard-Dunn T=925C

## VI. Glide builds walls, climb destroys them?!?

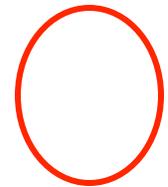


Continuous wall:  $(0,-1)$  to  $(0,1)$

Force on like dislocations:

**Glide only:**

- repulsive on side
- attractive at ends only

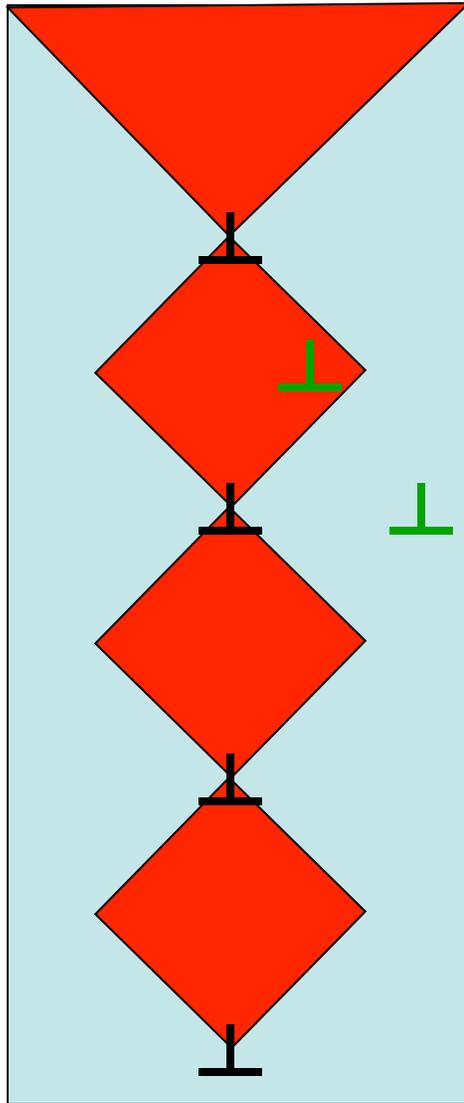


**Glide+climb:**

- repulsive everywhere ?!?



## VI. Discreteness Essential for Wall formation



Force on like (green) DL close to wall:

Repulsive: 

Attractive: 

**Growth from the side:**

only in the attractive diamonds,  
generated by discreteness

**Growth from the end:**

attractive funnel at end

## VI. Glide: Wall Formation: Simulation



1. The dislocations outside the attractive red diamonds of their neighbors fly out

2. Wall reassembles slowly, through

- attractive side-diamonds
- end-funnels

# VI. Glide: Walls in Field Theory: Reverse Diffusion

$$D\Delta^2\chi = b\partial_y\rho$$

$\chi$ : Airy Potential

$$f = b\tau = b\partial_x\partial_y\chi$$

$f$ : Peach-Kohler force

$$j = \rho v = \rho Bf$$

$$j = \rho Bb\partial_x\partial_y\chi + BT\partial_x\rho$$

Simplest extra term to capture discreteness

$$\partial_t\rho = ibB\rho_0k_x^2k_y\chi(k) + BTk_x^2\rho(k)$$

$$\partial_t\rho = \left[-Bb^2\rho_0\frac{k_x^2k_y^2}{Dk^4} + TBk_x^2\right]\rho(k)$$

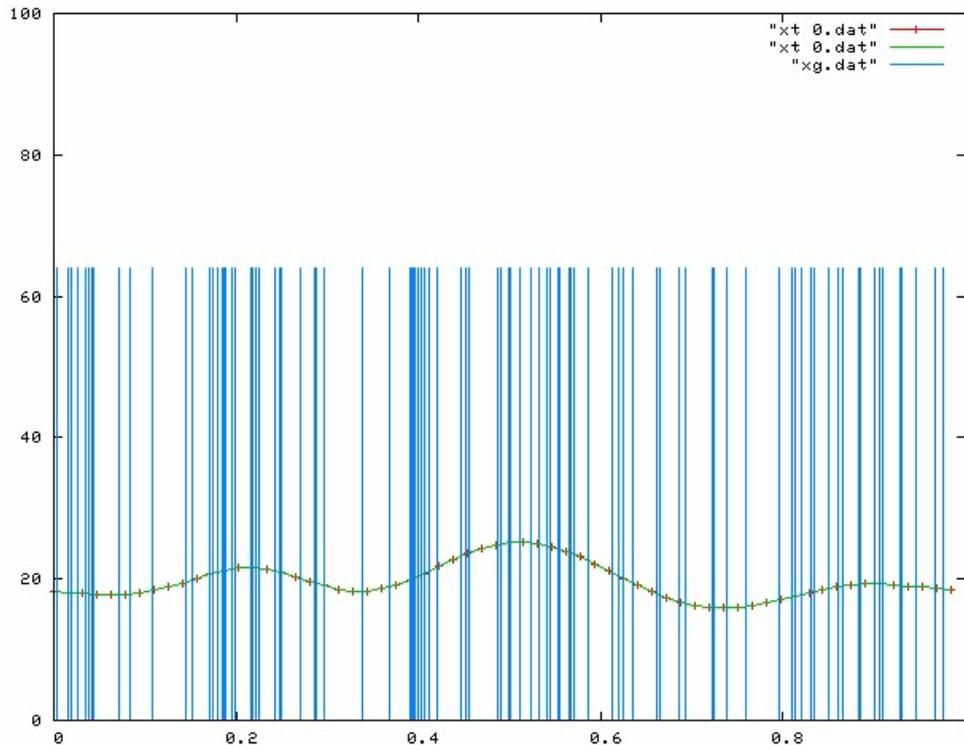
Without extra term:  
Fluctuations/walls do not grow

$$\partial_t\rho = BTk_x^2\rho(k)$$

$$k_y = 0$$

Extra term and  $k_y=0$ :  
Unstable at every  $k_x$ ,  
Larger  $k_x$  modes grow faster

# VI. Glide: Reverse Diffusion: Simulation



$$\partial_t \rho = \text{const.} \times \partial^2 \rho / \partial x^2$$

Positive curvature (maxima):  
always grow

Negative curvature (minima):  
always decay

**Walls indeed form!**

Where the initial conditions  
had maxima

Length scale for fluctuations of initial condition is  $\sim 1/\rho^{1/2}$   
So distance between walls  $d \sim 1/\rho^{1/2}$  : **Holt relation satisfied**

## VII. Glide+Climb: What Keeps Walls Together?

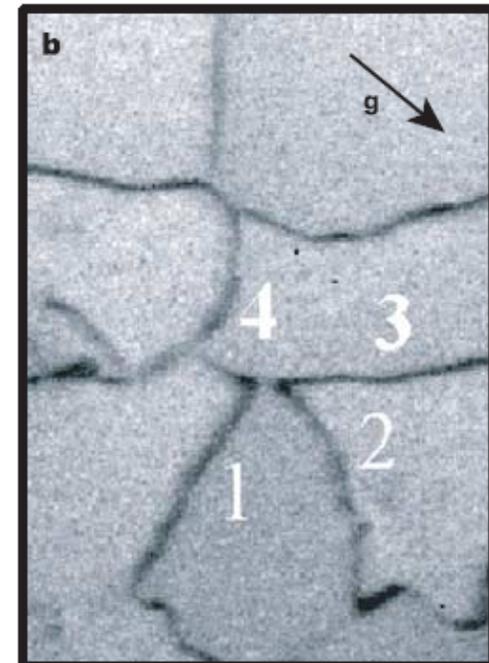
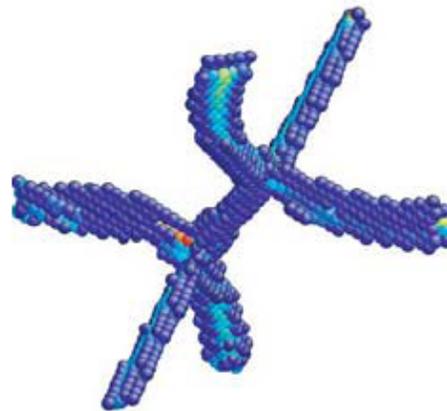
We saw in part I that climb is essential for wall formation.  
Yet, climb seems to allow walls flying apart.  
Why don't walls fly apart when climb is present?

3D: Junctions stabilize the patterns  
(entangled/zipped dislocation lines)

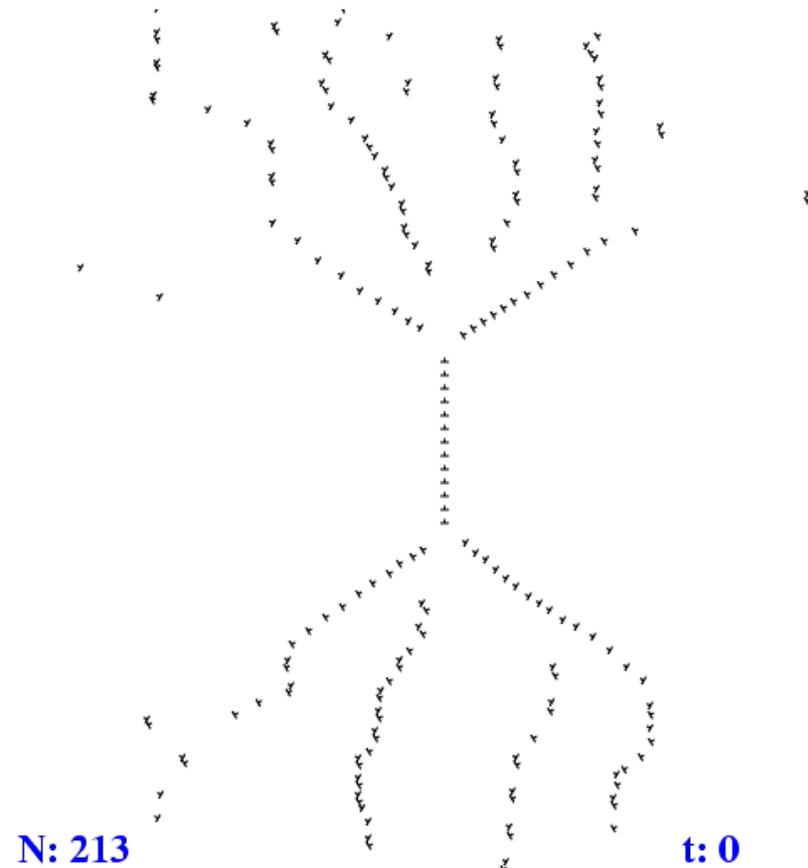
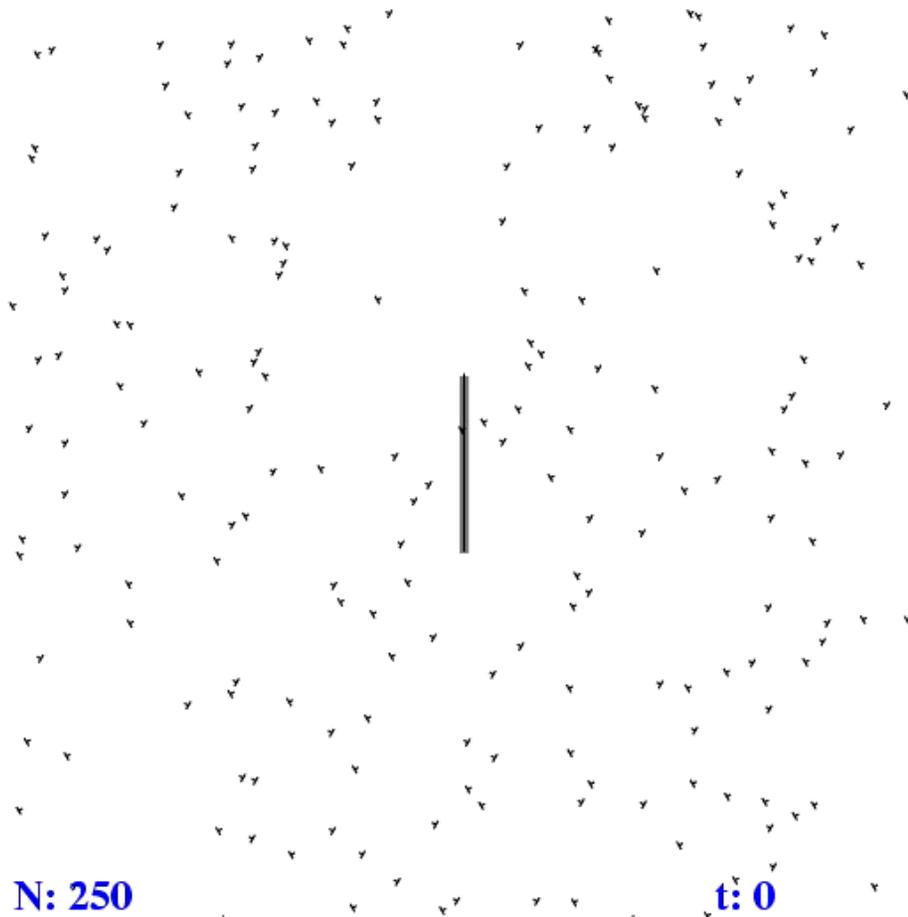
Bulatov (2006)

Kubin (next talk)

But: no junctions in 2D

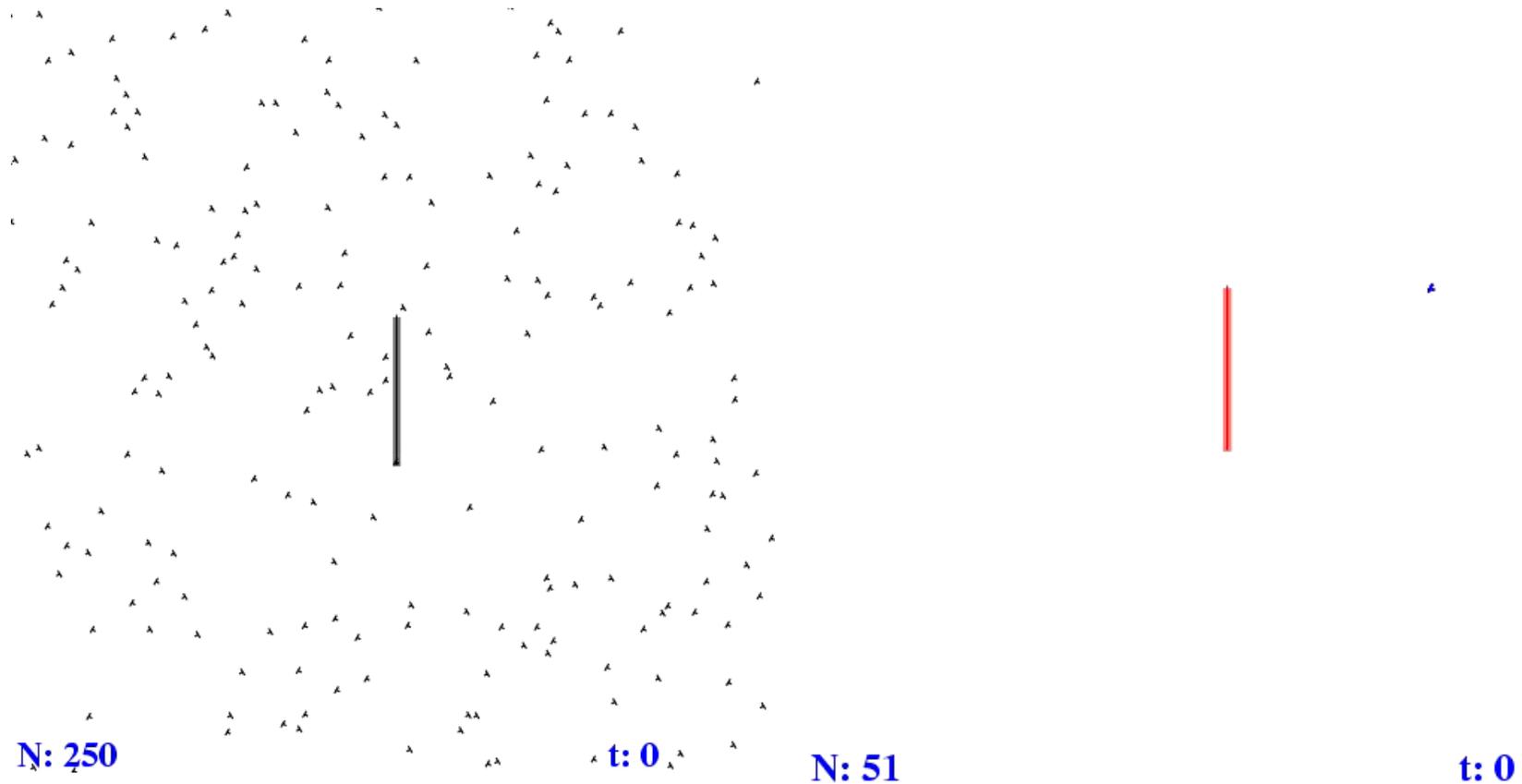


## VII. Anchors Stabilize Against Climb



**There are no junctions in 2D: what stabilizes structures?  
Anchors stabilize domain walls effectively against climb**

## VII. Anchors Stabilize Against Climb



**Anchors stabilize domain walls effectively against climb**

## VII. Ingredients of Wall Formation

- **Glide only + Polarized:**

Wall forms by attracting dislocations in “near field” and at end

- **Glide only + Neutral:**

Forces from opposite dislocations frustrate wall formation:  
10% in walls, 90% between

- **Glide + Climb:**

Climb allows opposite dislocations to annihilate  
Only like dislocations in walls survive

- **Anchors:**

Stabilize walls against flying apart by climb

# Summary

**Glide: Aging, Freezing ( $D_{\text{eff}}=0$ ): Dislocation Glass**

**Glide+Climb: Domain formation** – Expt.



**Glide+Climb: Coarsening:  $z=0.17$**  – Expt.



**Walls/Glide: Attraction from near field** – Sim' n



**Walls/Glide: Reverse diffusion field th.** – Sim' n



**Walls/G+C: Anchors stabilize walls** – Sim' n

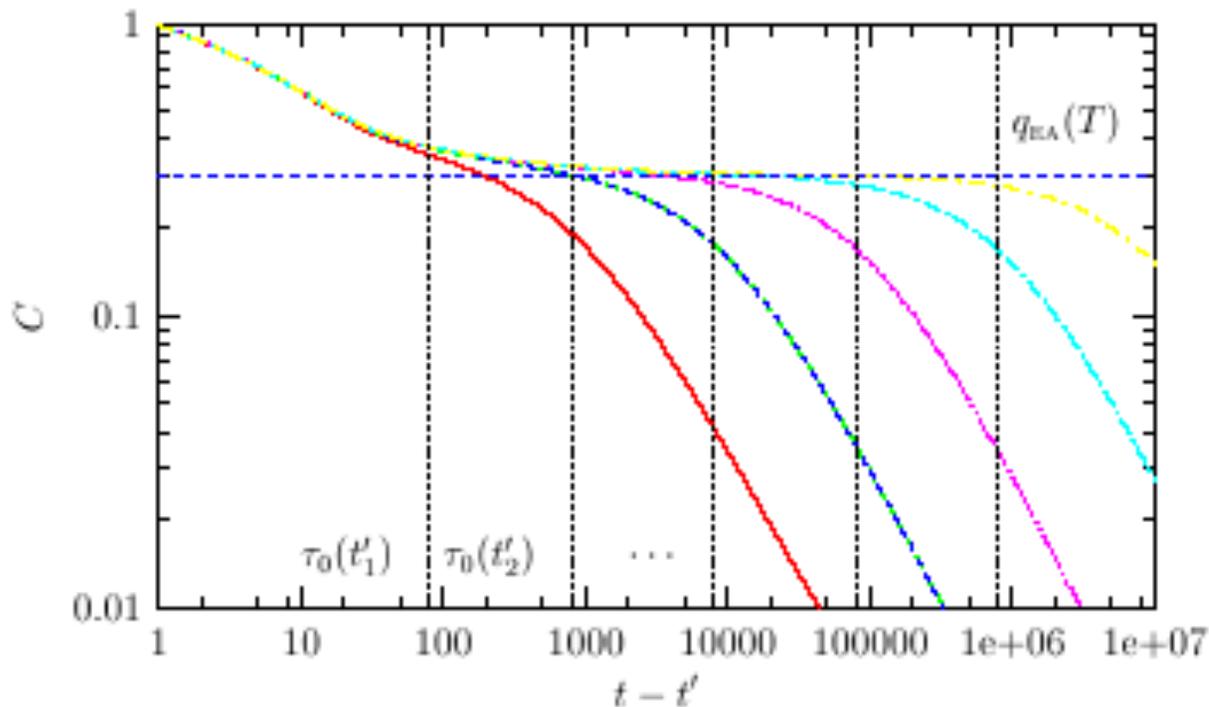


# Aging

## Connection to equilibrium:

Correlation function  $C(t, t')$  exhibits a plateau

$C(t, t')$  at plateau is the Edwards-Anderson (EA) order parameter

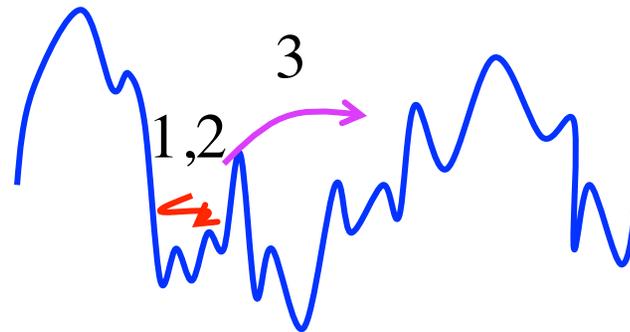


Increasing  $t_w$ , fixed  $T$

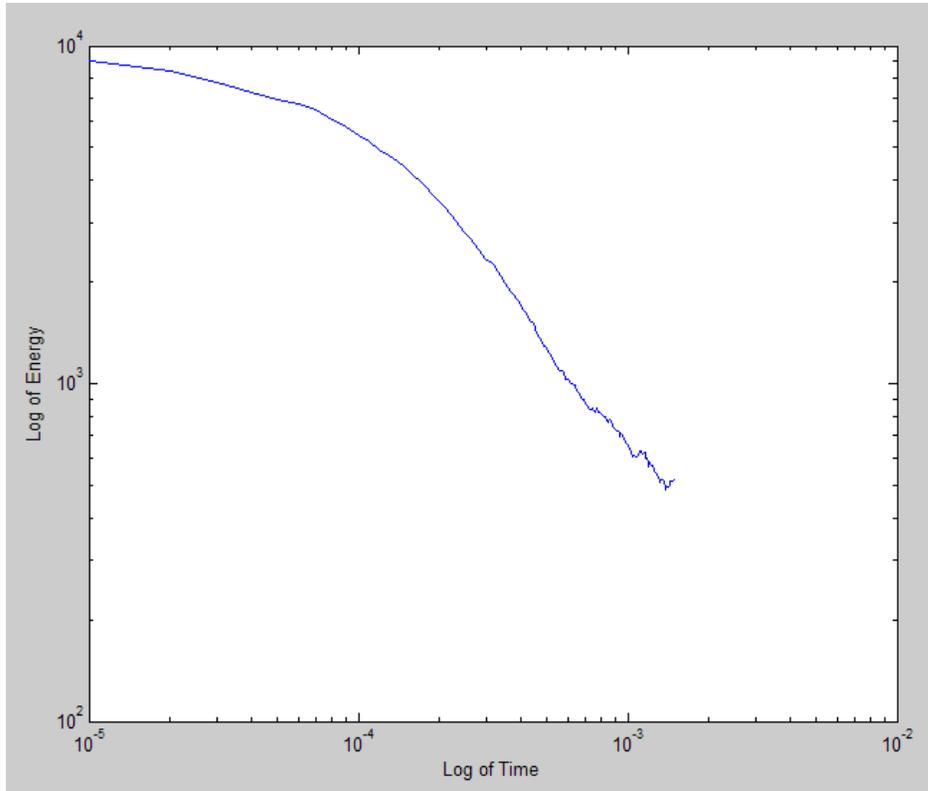
# Aging

## Stages:

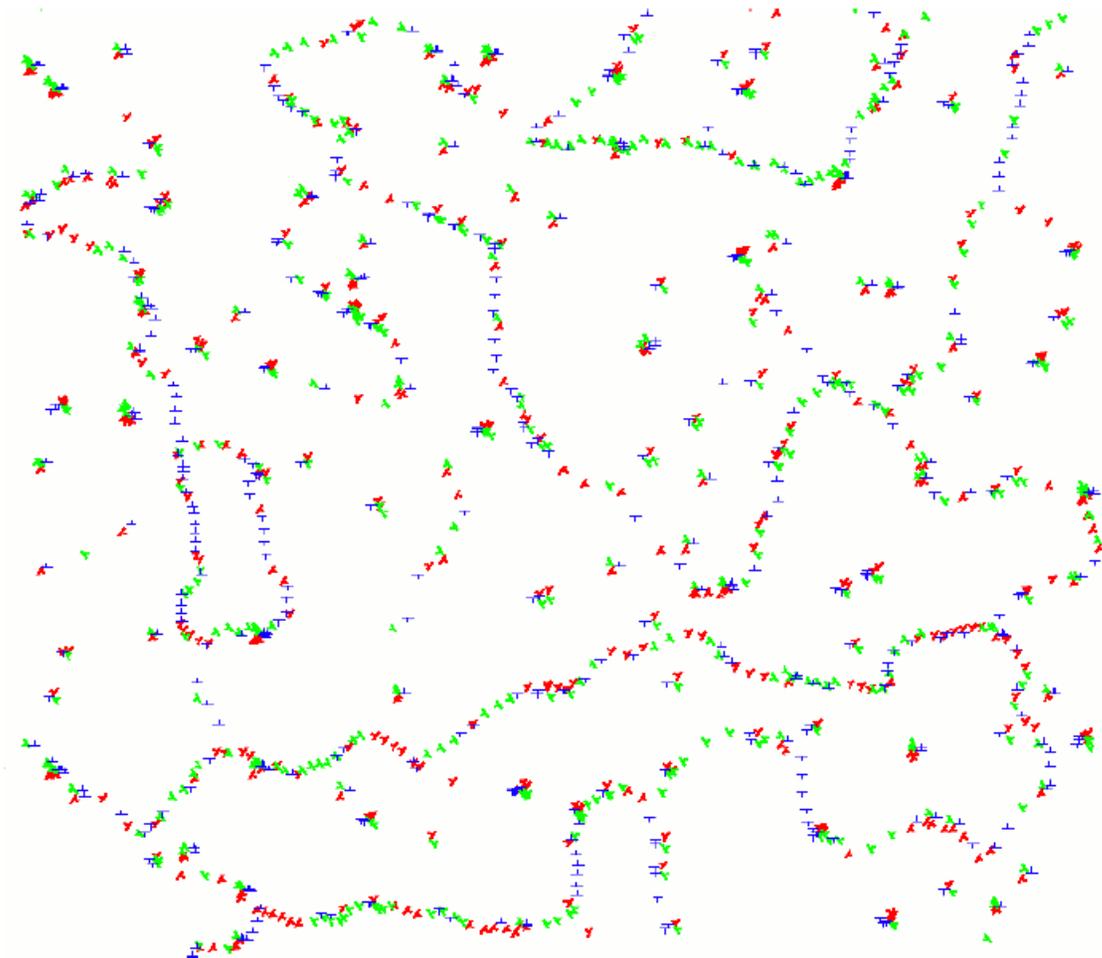
1.  $t < t_w$ : quick  $\beta$  relaxation: system expands from initial condition to explore boundaries of one well
2.  $t \sim t_w$ : plateau: system stuck within one well
3.  $t > t_w$ : slow  $\alpha$  relaxation: system escapes to other wells



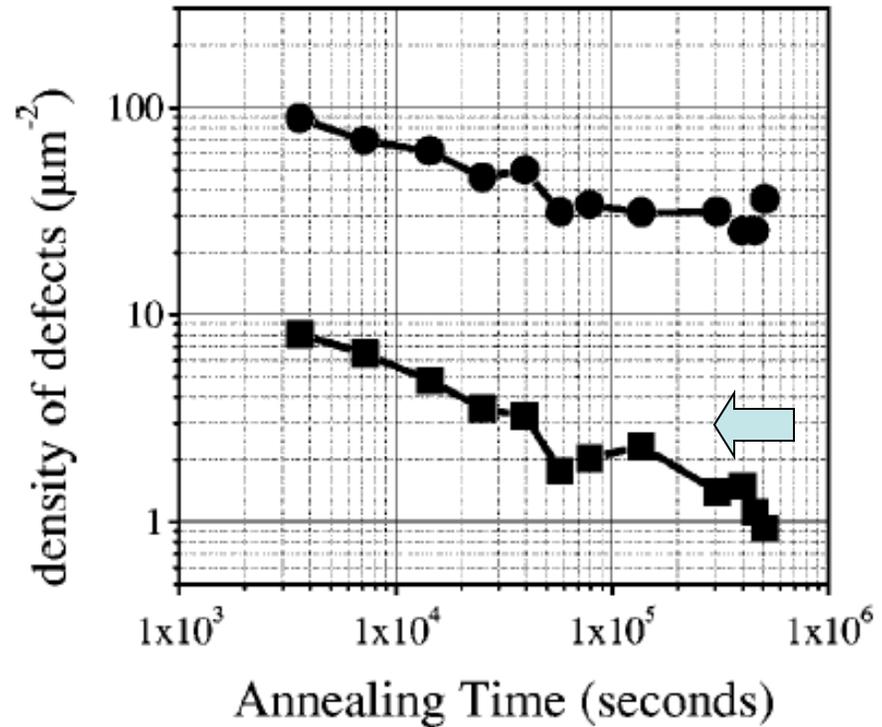
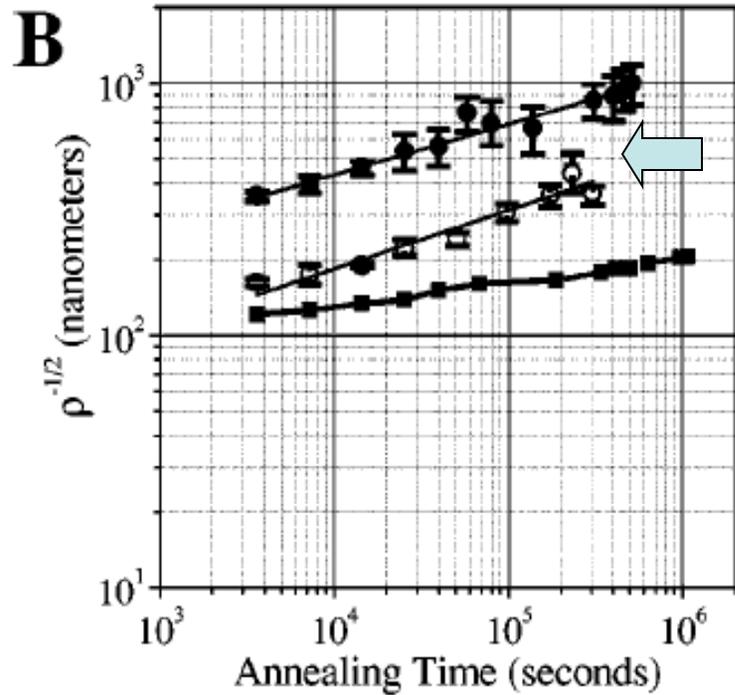
# Glide only, 1 axis, Polarized: No Glassy Dynamics



- No Aging
- $E(t) \sim t^{-1.4}$



# The Coarsening Exponent is 1/4



$$L \sim t^{1/4}$$

Chaikin, Huse, 2002