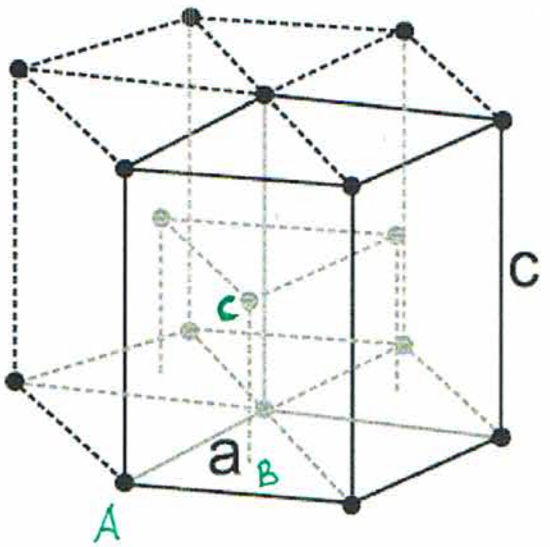
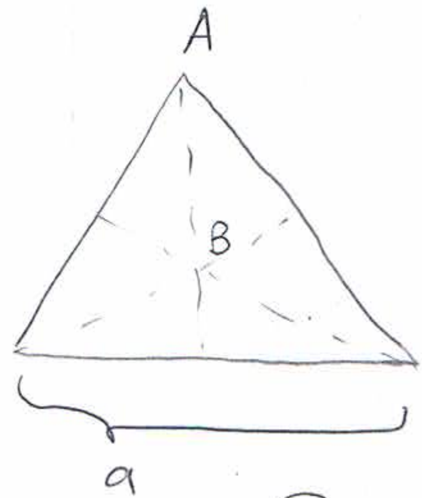
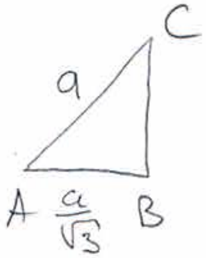
1.3





$$AB = \frac{2}{3} a \sin 60^\circ = \frac{2}{3} a \frac{\sqrt{3}}{2} = \frac{a\sqrt{3}}{3}$$



$$AC = a$$

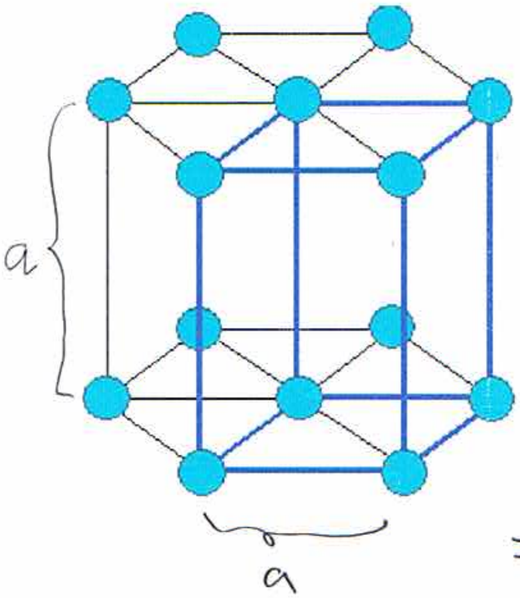
$$\Rightarrow BC = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} =$$

$$= a\sqrt{1 - \frac{1}{3}} = a\sqrt{\frac{2}{3}}$$

$$c = 2\,BC = a\sqrt{\frac{8}{3}}$$

$$\Rightarrow \frac{c}{a} = \left(\frac{8}{3}\right)^{\frac{1}{2}}$$

1.5 Hexagonal



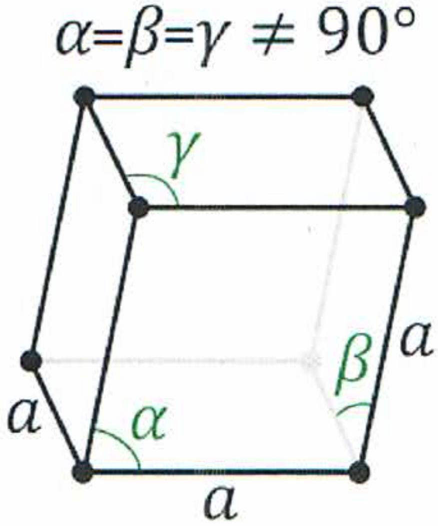
hexagonal $\Rightarrow$ Bravais lattice

$\Rightarrow$ 1 atom per unit cell

$$V_{unit\,cell} = a \cdot a \cdot \frac{\sqrt{3}}{2} \cdot a = a^3 \frac{\sqrt{3}}{2}$$

$$\Rightarrow n = \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3 \frac{\sqrt{3}}{2}} = \frac{\pi \cdot 4 \cdot 2}{3 \cdot 8 \cdot \sqrt{3}} = \frac{\pi}{3\sqrt{3}}$$

Rhombohedral



$V_{\text{unit cell}} = a^3 \sqrt{1 - 3\cos^2\alpha + 2\cos^3\alpha}$

(from wiki)

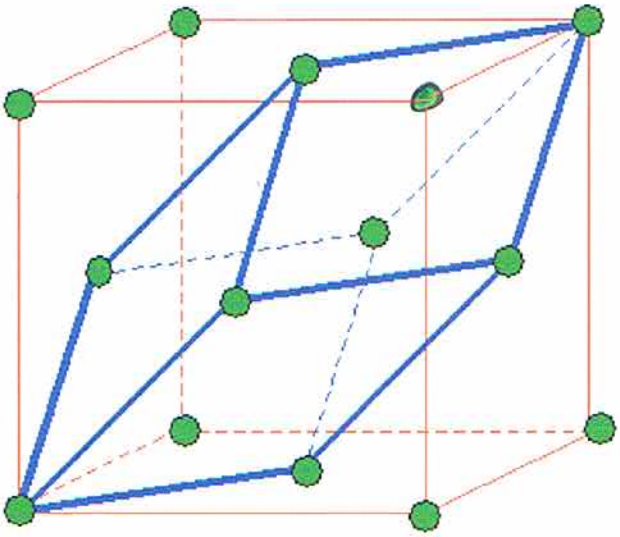
1 atom per unit cell

$R = \frac{a}{2}$

$$\Rightarrow \quad n = \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3\sqrt{1 - 3\cos^2\alpha + 2\cos^3\alpha}} =$$

$$= \frac{\pi}{6\sqrt{1 - 3\cos^2\alpha + 2\cos^3\alpha}}$$

1.7



Unit cell - 1 atom per cell

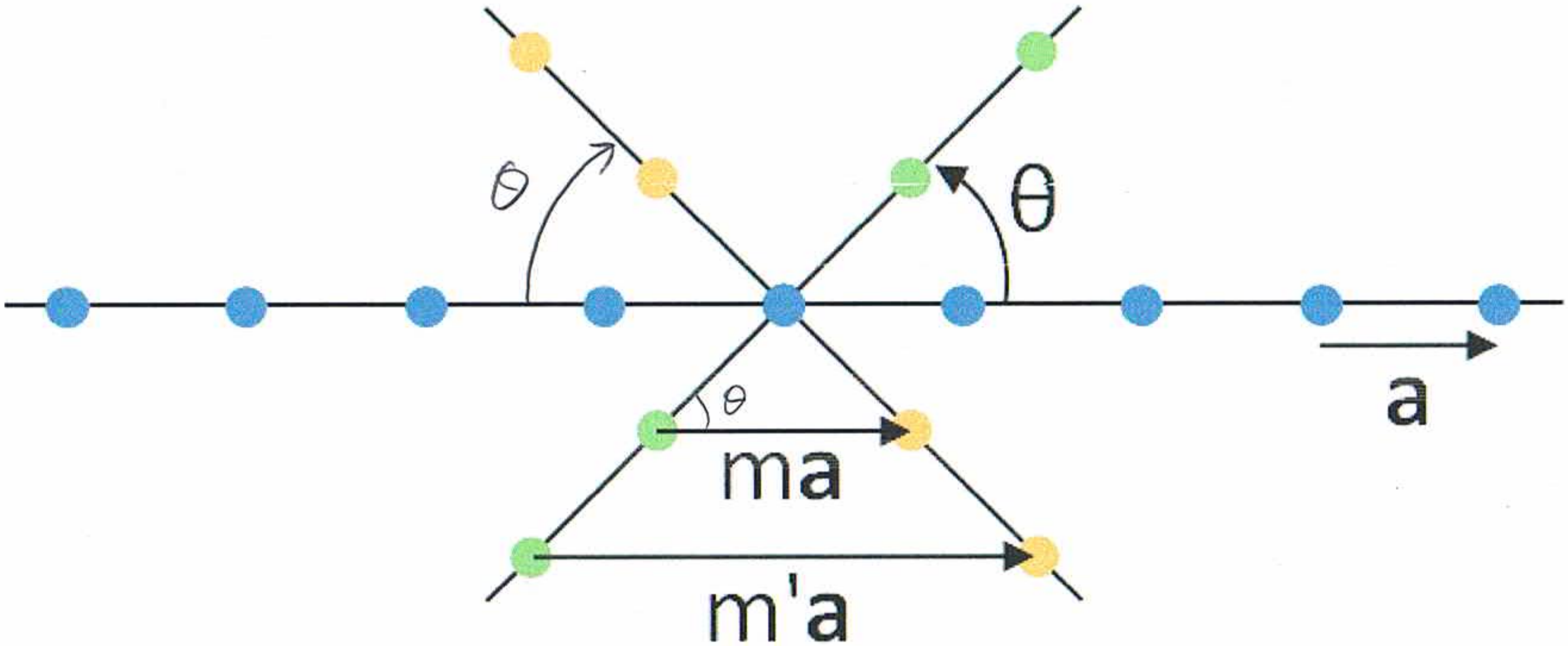
Conventional cell :

$8 \cdot \frac{1}{8}$ - corners

$6 \cdot \frac{1}{2}$ - sides

$\Rightarrow$ 4 atoms per cell

1.8 (More solutions at wiki $\Rightarrow$ Crystallographic restriction theorem)



$\theta = \frac{2\pi}{n}$ Because of the periodicity m - integer

from the 

triangle $2a \cos\theta = ma$ $\Rightarrow$

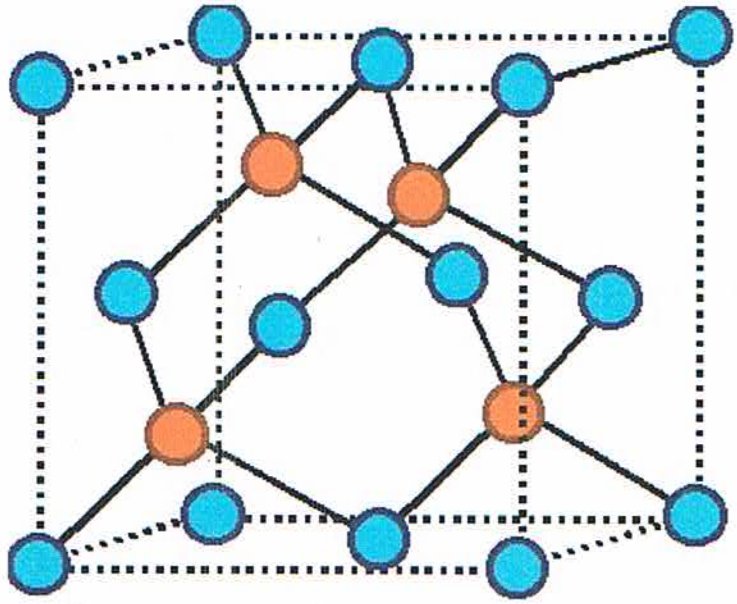
$$\Rightarrow 2\cos\frac{2\pi}{n} = m \quad *$$

m, n - integers

The only n, m pairs that satisfy equation $(*)$ are: $\Rightarrow$ $n = 1, 2, 3, 4, 6$

$m = -1, 0, 1$

1.13



The diamond lattice contains two face centered cubic lattice so that the total number of atoms per unit cell equals twice that of the face centered lattice. The atoms touch along the body diagonal, where two atoms are one quarter of the body diagonal apart or $\sqrt{3}a/4$.

$$\Rightarrow r = \frac{\sqrt{3}a}{8}$$

$$n = \frac{\frac{4}{3}\pi\left(\frac{\sqrt{3}a}{8}\right)^3 \cdot 8}{a^3} = \frac{\pi\sqrt{3}}{16}$$

1.14

$$E = N\frac{A}{R^n} - N\frac{\alpha e^2}{4\pi\epsilon_0 R}$$

a) $$\frac{\partial E}{\partial R} = -nN\frac{A}{R^{n+1}} + N\frac{\alpha e^2}{4\pi\epsilon_0 R^2} =$$

$$= -nNA\,R^{-n-1} + N\frac{\alpha e^2}{4\pi\epsilon_0}R^{-2} = 0$$

$$\Rightarrow R_0^{\,n-1} = \frac{4\pi\epsilon_0 A}{\alpha e^2}\,n$$

b) $$E(R_0) = \frac{N}{R_0}\left(\frac{A}{R_0^{\,n-1}} - \frac{\alpha e^2}{4\pi\epsilon_0}\right) =$$

$$= \frac{N}{R_0}\left(\frac{\alpha e^2 A}{4\pi\epsilon_0 A\,n} - \frac{\alpha e^2}{4\pi\epsilon_0}\right) =$$

$$= -\frac{N}{R_0}\frac{\alpha e^2}{4\pi\epsilon_0}\left(1-\frac{1}{n}\right)$$

c) $\alpha = 1.75$ $\qquad\qquad$ $\epsilon_0 = 8.85\ 10^{-12}$ F/m

$E_0 = -7.95$ eV/molecule

$R_0 = 5.63$ Å$/2$

$$-7.95\cdot 1.6\cdot 10^{-19} = \left(\frac{1}{n} - 1\right)\frac{2\cdot 1.75\,(1.6\ 10^{-19})^2}{5.63\cdot 10^{-10}\cdot 4\pi\cdot 8.85\cdot 10}$$

$$\Rightarrow \frac{1}{n} - 1 = -0.88 \qquad \Rightarrow \qquad n \approx 9$$