PHY140 HW6 CHAPTER 4 OMAR BOOK.

Q4 Assuming e^-s are classical gas, $T = 300K$

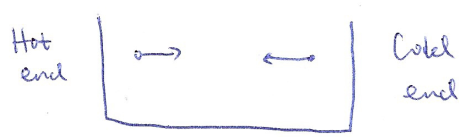
$$\frac{1}{2} m^* V_{rms}^2 = \frac{3}{2} kT$$

$$\Rightarrow V_{rms} = \sqrt{\frac{3kT}{m^*}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}} \ m/s$$

$$= 1.17 \times 10^5 \ m/s$$

Compared to $V_F = 1.57 \times 10^6 \ m/s$, V_{rms} is much smaller. This indicates the conduction e^-s are not free and subject to strong interactions (e.g. Coulomb forces) in the metal. Simple classical theory is not applicable.

Q5



* Electrons move randomly, hotter ones vibrates more while colder ones vibrates less. On average, there are <u>equal</u> amount of e^-s moving in opposite directions. Therefore there is <u>no</u> net particle flux.

* Hotter e^-s moving to the colder side does carry more energy, So there will be a net energy flow.

Problem 1.

a) concentration of conduction e^- :

Each Cu gives one conduction e^-

So # concentration of e^- is

$$n = \frac{\rho_m}{m_{Cu}} = \frac{8.95\,g/cm^3}{63.546\,g/mol} = \frac{6.02\times10^{23}\times 8.95\times10^{6}}{63.546}\,m^{-3} = \underline{\underline{8.48\times10^{28}\,m^{-3}}}$$

b) The mean free time τ :

$$\tau = \frac{m^*}{ne^2}\cdot\frac{1}{\rho} = \frac{9.1\times10^{-31}}{8.48\times10^{28}\cdot(1.6\times10^{-19})^2}\cdot\frac{1}{1.55\times10^{-8}} = \underline{\underline{2.7\times10^{-14}\,s}}.$$

c) The Fermi energy E_F

$$E_F = \left(\frac{\hbar^2}{2m^*}\right)(3\pi^2 n)^{2/3} = \frac{(1.05\times10^{-34})^2}{2\times9.1\times10^{-31}}\cdot\left(3\pi^2\cdot8.48\times10^{28}\right)^{2/3}$$

$$= 1.12\times10^{-18}\,J = \underline{\underline{7\,eV}}$$

d) The Fermi velocity V_F

$$V_F = \sqrt{\frac{2E_F}{m^*}} = \sqrt{\frac{2\times1.12\times10^{-18}}{9.1\times10^{-31}}} = \underline{\underline{1.57\times10^{6}\,m/s}}$$

e) Mean free path l_F :

$$l_F = V_F\cdot\tau = 1.57\times10^{6}\times2.7\times10^{-14} = 4.2\times10^{-8}\,m \simeq \underline{\underline{420\,\mathring{A}}}$$

Recall Cu lattice spacing $\sim 4\,\mathring{A}$

So each e^- travels ~ 100 lattice sites between two scatterings $\Rightarrow$ Cu is a good conductor.

Problem 6

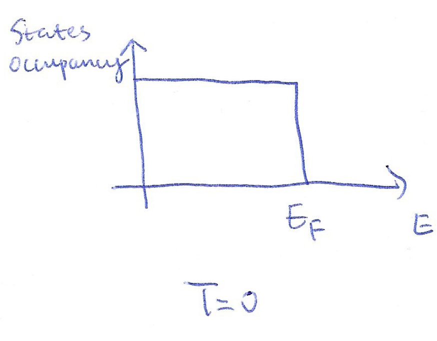
Use Table 4.1 , $T_F = \frac{E_F}{k_B}$, $T = 300\,K$

	E_F / eV	T_F / K	T/T_F
Cu	7	8×10^4	3.8×10^{-3}
Na	3.1	3.6×10^4	8.3×10^{-3}
Ag	5.5	6.4×10^4	4.6×10^{-3}

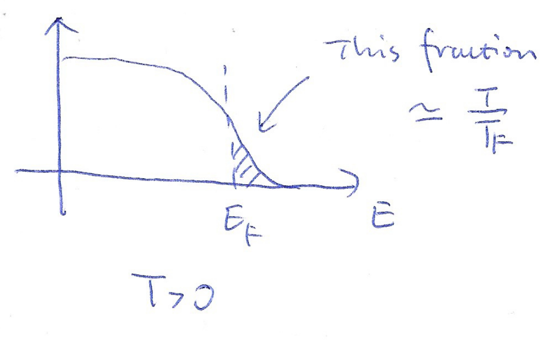
Problem 7

Fraction of e^-'s excited above Fermi level

$f \simeq \frac{k_B T}{E_F} = \frac{T}{T_F}$, i.e. just last column of table above.



$\Rightarrow$



Problem 8

Electronic contribution to specific heat

$$C_e = \frac{\pi^2}{2} \frac{T}{T_F} \cdot R \quad \text{where } T_F = 8 \times 10^4 \text{ K}.$$

Lattice specific heat is given by the <u>Debye Model</u>

$$C_l = \frac{\partial U}{\partial T} = 9 N_A k_B \left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta_D/T} x^4 \frac{e^x}{(e^x-1)^2}\,dx$$

and $\Theta_D = 343$ K for Cu.

At low T ($T \to 0$, $T \ll \Theta_D$)

$$C_l \simeq \frac{12\pi^4}{5} N k_B \left(\frac{T}{\Theta_D}\right)^3$$

At high T ($T \gg \Theta_D$), $\quad C_l \simeq 3 N k_B$.

In intermediate T, need to evaluate the above integral.

Calculate the ratio $C_e/C_l = \frac{\pi^2}{18} \cdot \frac{\Theta_D^3}{T_F} \cdot \frac{1}{T^2} \cdot \frac{1}{\int_0^{\Theta_D/T} x^4 \frac{e^x}{(e^x-1)^2}\,dx}$

↓

Tip: Use Wolfram alpha

type:

Integrate

(x^4)*(e^x)/(e^x-1)^2

from 0 to 343/T

T/K	Ce/Cl
0.3	118
4	0.67
20	0.027
77	0.0036
300	0.0065