3.11

$$U_n = A e^{i(q x_n - \omega t)} \qquad (\text{eq } 3.45)$$

$$q \to iq \qquad U_n = A e^{i(iq x_n - \omega t)} =$$

$$= A e^{-q x_n} e^{-i\omega t} \qquad x_n = n a$$

$$M \ddot{U}_n = -\alpha \left( 2U_n - U_{n+1} - U_{n-1} \right)$$

$$M(-i\omega)^2 A e^{-q x_n} e^{-i\omega t} = -\alpha \Big[ 2 A e^{-q x_n} e^{-i\omega t} -$$

$$- A e^{-q x_{n+1}} e^{-i\omega t} - A e^{-q x_{n-1}} e^{-i\omega t} \Big]$$

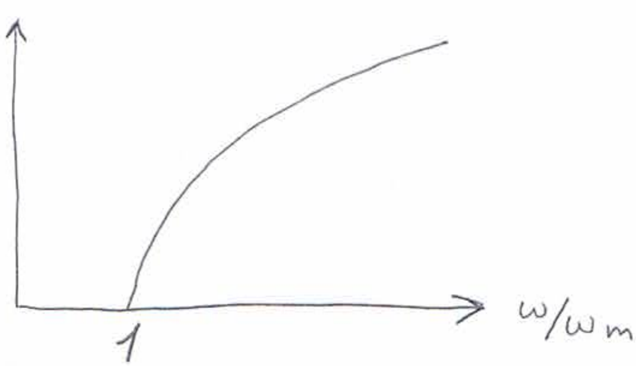
$$M\omega^2 e^{-qna} = \alpha \left[ 2 e^{-qna} - e^{-qna} e^{-qa} - e^{-qna} e^{qa} \right]$$

$$\frac{M\omega^2}{\alpha} = 2 - e^{-qa} - e^{qa} = 2 - 2\cosh(qa)$$

$$qa = \cosh^{-1}\left(1 - \frac{M\omega^2}{2\alpha}\right) = \cosh^{-1}\left(1 - \frac{\omega^2}{\omega_m^2}\right)$$

$$I = |U_n|^2 = A^2 e^{-\underbrace{2qa}_{}n}$$

↑ Attenuation coefficient



3.12

$$(\text{eq } 3.59) \quad \begin{bmatrix} 2\alpha - M_1\omega^2 & -2\alpha\cos(qa) \\ -2\alpha\cos(qa) & 2\alpha - M_2\omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0$$

$$\begin{matrix} q=0 \\ \omega=0 \end{matrix} \Rightarrow 2\alpha A_1 - 2\alpha A_2 = 0$$

$$\Rightarrow \boxed{A_1 = A} \qquad (\text{eq } 3.62)$$

$$q=0$$

$$\omega = \left[2\alpha\left(\frac{1}{M_1} + \frac{1}{M_2}\right)\right]^{\frac{1}{2}}$$

$$\left[2\alpha - 2\alpha M_1\left(\frac{1}{M_1} + \frac{1}{M^2}\right)\right] A_1 - 2\alpha A_2 = 0$$

$$\left[1 - 1 - \frac{M_1}{M_2}\right] A_1 - A_2 = 0$$

$$\Rightarrow M_1 A_1 + M_2 A_2 = 0 \qquad (\text{eq } 3.63)$$

3.13

$\rho = 2.18\ g/cm^2$

Transverse : $\omega = \omega_t = 3.08 \cdot 10^{13}$ rad/s $=$

$= 4.92 \cdot 10^{12}$ Hz

Longitudinal: $\omega = \omega_t = 5 \cdot 10^{13}$ rad/s $=$

$=$

$M_1 = 22.99$ amu $= 3.82 \cdot 10^{-26}$ kg

$M_2 = 35.45$ amu $= 5.89\ 10^{-26}$ kg

Optical mode:

$$\omega = \sqrt{2\alpha\left(\frac{1}{M_1} + \frac{1}{M_2}\right)}$$

$$\omega^2 = 2\alpha\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

$$\alpha = \frac{\omega^2}{2\left(\frac{1}{M_1} + \frac{1}{M_2}\right)}$$

$$\alpha = \frac{(3.08\ 10^{13}\ Hz)^2}{2\left(\frac{1}{3.82 \cdot 10^{-26} kg} + \frac{1}{5.89 \cdot 10^{-26}}\right)} \approx 10.99\ \frac{kg}{s}$$

$$\alpha = a Y \Rightarrow Y = \frac{\alpha}{a} \approx \frac{10.99\ kg/s^2}{5.64 \cdot 10^{-10} m} \approx$$

$$\approx 1.95^{10} \cdot 10^8\ kg/m \cdot s^2 = 1.95\ 10^{10}\ Pa$$

Longitudinal:

$$\alpha = 28.96 \ {}^{kg}/_{s^2} \qquad Y = 5.14 \cdot 10^{10} \ Pa$$

$$\Rightarrow \quad v_{s_e} = \sqrt{\frac{Y}{\rho}} \approx \sqrt{\frac{1.95 \cdot 10^{10}}{2.18 \cdot 10^{3}}} \approx 2990 \ {}^{m}/_{s}$$

$$v_{s_t} = \sqrt{\frac{Y}{\rho}} \approx \sqrt{\frac{5.14 \cdot 10^{10}}{2.18 \cdot 10^{3}}} \approx 4855 \ {}^{m}/_{s}$$